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Diversity Order and Degrees of Freedom for Cooperative Communication Networks

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이 논문을 공학박사 학위논문으로 제출함.

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Diversity Order and Degrees of Freedom for Cooperative Communication Networks

Presented to the Graduate School of Seoul National University in Partial Fulfillment of the Requirements for

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by

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Abstract

Diversity Order and Degrees of Freedom for Cooperative Communication Networks

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This dissertation contains the following three contributions to the interesting research topics on diversity techniques and interference alignment (IA) for cooperative communication networks.

- Relay on-off threshold (ROT) for non-orthogonal decode and forward (NDF) protocol with distributed orthogonal space-time block codes (DOSTBCs)
  - Calculate the optimal ROT for NDF protocol with DOSTBCs in high signal to noise power ratio (SNR) region.
  - Propose suboptimal ROT for NDF protocol with DOSTBCs in low SNR region.
  - Analyze the diversity order of the proposed scheme.
- New IA schemes aided by relays for quasi-static $M \times 2$ X channel
  - Propose IA scheme aided by one full-duplex relay.

  - Propose IA scheme aided by two half-duplex relay.
  - Prove that the proposed IA schemes achieve the maximum degrees of freedom (DoF) for quasi-static $M \times 2$ X channel.
- Selection diversity on the IA for multi-input and multi-output (MIMO) interference channel
  - Propose the selection criterion of beamforming matrices of IA for MIMO interference channel.
  - Analyze the diversity order of the proposed scheme.

First, we construct the DOSTBCs using source and relay in the cooperative communication networks. In order to decode the DOSTBCs, the destination uses the linear combining (LC) decoding scheme. In this system models, the symbol error rate (SER) is formulated and the ROT is calculated to minimize the SER. It is proved that the full diversity order of NDF protocol can be achieved by using the relay on-off scheme with the optimal threshold.

In the second part of this dissertation, two new IA schemes aided by relays for quasi-static $M \times 2$ X channel are proposed. The first proposed scheme uses one full-duplex relay and it can achieves the maximum DoF. However, at the full-duplex relay, the transmit signal can be strong self-interference, called echo and thus it is difficult to implement practically. To resolve this problem, at the second proposed IA scheme, two half-duplex relays are used and it is proved that the second proposed IA scheme can also achieve the maximum DoF of $M \times 2$ X channel.

Finally, the selection scheme for IA is proposed for MIMO interference channel. Most of IA schemes are focused on DoF but there is only a few research results for diversity order which is a crucial measure of reliability. Therefore, we propose a selection criterion to minimize SER and analyze the diversity order of the proposed scheme in the MIMO interference channel.

Keywords: Cooperative communication networks, degrees of freedom (DoF), distributed orthogonal space-time block codes (DOSTBCs), diversity order, interference alignment (IA), interference channel, relay, selection diversity, X channel.

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Chapter 1. Introduction

1.1. Background

The recent advancement in wireless communication systems has increased their throughput and reliability. As a result, mobile devices become more popular than wired devices. However, as the market of the mobile contents becomes bigger and various services are introduced widely, throughput and reliability become more important research topics. Even though new services which require high data transmission rate are released, for the reliability, the data rate should be restricted and thus the services cannot be provided properly. In fact, this dilemma has led to the advance of wireless communication technology.

Multi-input and multi-output (MIMO) as a new technology has drawn great attention because it provides significant increases in data throughput and reliability without additional bandwidth or increase in transmit power. It achieves the enhancement of performance by spreading the total transmit power over the antennas to obtain multiplexing gain that improves the spectral efficiency and to obtain diversity gain that improves the reliability. Due to these properties, MIMO technology is an important part of modern wireless communication systems.

In the MIMO technology, two kinds of gains have been investigated. One is the diversity gain which is related to the reliability and the other is multiplexing gain which is related to the throughput. Sometimes, the multiplexing gain may be called degrees of freedom (DoF). Cheng and Tse [1] revealed the tradeoff of these two gains when the channel state information (CSI) is not available at the transmitter and proved the existence of the code to achieve so called the fundamental tradeoff.

In general, transmitted signals experience deep fading distortion in wireless communications. To overcome this fading, diversity technique has been adopted, which is a good solution to obtain the diversity gain. Space-time codes (STCs) are the most famous scheme to obtain diversity gain without CSI. Tarokh et al. [2] proposed the design criterion for STCs and Alamouti [3] designed a simple space-time block code (STBC). After that, many STBCs are proposed such as orthogonal and quasi-orthogonal STBCs (OSTBCs, QOSTBCs) and coordinated interleaving STBCs (CISTBCs) and their performances have been analyzed from the viewpoint of diversity gain.

Through MIMO technology such as STCs, the reliability can surely be enhanced much more than before. However, the deployment of multiple transmit and receive antennas in a limited space is difficult and costly. This makes researchers to consider using relays to create a virtual MIMO channels for wireless communication systems, called the cooperative communication, which is considered as a core technology in the next generation wireless communication systems.

The protocols of the cooperative communications are classified according to the operation of relay. In orthogonal transmission (OT) protocol, the relay keeps silent when the transmitter transmits its signal but in non-orthogonal transmission (NT) protocols, the relay transmits while the transmitter transmits. And in amplify-and-forward (AF) protocol, the relay amplifies the received signal and just forward it to the destination. In decode-and-forward (DF) protocol, the relay decodes the received signal and forwards it to the destination.

In general, it is assumed that the relay does not generate new information but it transmits the replica of information of transmitters and thus the cooperative communication is mainly focused on obtaining the diversity gain. This technique is called the cooperative diversity technique. Since the relay can be thought as a virtual antenna for the source to destination, it can be used to design the distributed STCs (DSTCs) to obtain the cooperative diversity [4].

Through the MIMO technology and cooperative communications, the reliability and throughput can be improved greatly for pear-to-pear (P2P) communications. However, in the wireless communication environment where many mobile devices exist, these techniques may cause strong interference.

In addition, due to introduction of small cell technology such as WiFi and femto-cell and so on, the radius of a cell becomes smaller and it makes the cell boundary area wider. Usually, the problems related to interference occur in the cell boundary and thus the small cell technology makes the problem of interference severe.

Even though the research on interference management has done since

1980's, the optimal scheme has not been introduced yet. Treating the interference as noise, decoding the interference, and orthogonalization are classical techniques for the interference management. However, the first scheme does not guarantee the reliability under the strong interference and the second one requires too much decoding complexity. The last one guarantees the reliability and low complexity but its throughput becomes low.

Recently, interference alignment (IA) has attracted a great attention as a good solution to resolve the drawbacks of the previous research. Cadambe and Jafar [5] proposed the IA scheme for K-user interference channel, which achieves the maximum DoF. Similarly, they proposed IA scheme for X channel in [6].

However, the IA schemes in [5, 6] have several problems. The first is that the time varying channel is assumed and is also assumed that the perfect global CSI is available at the transmitter, which requires lots of feedback information. Thus these two assumptions cannot simultaneously be accepted. And thus, recently, IA is implemented by using multiple antennas or multiple carriers under quasi-static channel assumptions.

In fact, it is limited by the size of device to use multiple antennas and thus there are some research results to use relay [7, 8, 9], which are called relay-aided IA schemes. Since the relay-aided IA schemes are based on the quasi-static channel, they are more practical than the IA schemes in the time-varying channel [5, 6].

The other problem is that IA scheme is only focused on the throughput which is measured by DoF and the reliability is not considered seriously. Since the reliability is also important in the wireless communications, the enhancement of the error performance of IA can be a good research topic.

1.2. Overview of Dissertation

This dissertation is organized as follows. In Chapter 2, some preliminaries of diversity techniques and IA schemes are explained.

In Chapter 3, the relay on-off threshold (ROT) for non-orthogonal DF (NDF) protocol with OSTBCs and linear combining (LC) decoding are derived. It is assumed that the source antenna switching (SAS) [10] is used in this chapter. And for M-QAM, it is proved that the proposed scheme can achieve full diversity order for NDF protocol with SAS.

In Chapter 4, the new IA schemes aided by one full-duplex relay and two half-duplex relays are proposed and the proposed IA schemes can achieve the maximum DoF for $M \times 2$ X channel. Also, for $2 \times M$ X channel, it is proved that the proposed schemes achieve the maximum DoF by reciprocity.

In Chapter 5, we propose a selection method of the beamforming matrices for IA and prove that the more diversity gain can be achieved than the IA without selection. Especially, for 3-user MIMO interference channel, when each node has two antennas, it is proved that diversity order two can be achieved by using the proposed selection scheme. And for more than two antennas, some analysis and numerical results are also given.

Finally, in Chapter 6, the concluding remarks are addressed.

1.3. Terms and Notations

In this section, we list the terms and notations used in this dissertation as in Tables 1.1 and 1.2.

Table 1.1: Terms used in the dissertation.

Term	Meaning
AF	amplify-and-forward
AWGN	additive white Gaussian noise
BER	bit error rate
CDF	cumulative distributed function
CSI	channel state information
DF	decode-and-forward
DMT	tradeoff between multiplexing and diversity gain
DOSTBC	distributed orthogonal space-time block code (coding)
DSTC	distributed space-time code (coding)
IA	interference alignment
LC	linear combining
MIMO	multiple-input and multiple-output
ML	maximum likelihood
NT, OT	non-orthogonal transmission, orthogonal transmission
ODF, NDF	orthogonal, non-orthogonal DF
OSTBC	orthogonal space-time block code (coding)
PDF	probability density function
PEP	pairwise error probability
QAM	quadrature amplitude modulation
ROT	relay on-off threshold
SAS	source antenna switching
S, D, R	source, destination, relay
SER	symbol error rate
SNR	signal to noise power ratio
STBC	space-time block code (coding)
STC	space-time code (coding)

Table 1.2: Notations used in the dissertation.

Notation	Meaning
$\mathcal{C}$	set of complex number
$f_X(\cdot)$	PDF of random variable X
$F_X(\cdot)$	CDF of random variable X
$\mathcal{K}_{\mathbf{x}}$	covariance matrix of vector $\mathbf{x}$
$X \sim CN(0, \sigma^2)$	X is a complex normal random variable with zero mean and variance $\sigma^2/2$ in both real and imaginary parts, respectively
$\mathbb{Z}$	set of integers
$\mathbf{0}_n$, $\mathbf{I}_n$	zero matrix and the identity matrix of size n
$(\cdot)^{\mathrm{T}}$	transpose of a matrix or a vector
$(\cdot)^{\dagger}$	conjugate transpose of a matrix or a vector
$\|\cdot\|$	Frobenius norm of a matrix or a vector
$\lvert\cdot\rvert$	modulus of complex number
$\mathcal{R}\{\cdot\}$	real part of complex number
$\mathcal{I}\{\cdot\}$	imaginary part of complex number
$(\cdot)^*$	complex conjugate of complex number

Chapter 2. Diversity Techniques and Interference Alignment

In this chapter, we review MIMO, cooperative communications, and IA. MIMO communications can be used for two different goals. One is to obtain higher data transmission rate and the other is to achieve higher reliability. Diversity techniques such as STCs and selection diversity are focused on the high reliability. Since Alamouti codes [3] were introduced, STCs, which are classified as transmit diversity technique in general, have drawn great interests. However, due to the limitation of the multiple-antenna implementation in a mobile node, the virtual multiple-antenna scheme is considered. Cooperative communications are well-known virtual multiple-antenna schemes which use relays as the virtual antennas for the source to the destination.

In fact, STCs and cooperative communications are diversity techniques for the P2P communications and they can transmit strong desired signals to the destination node. But the strong desired signals can be a strong interference for the other nodes. Therefore, without interference management, the reliable communication is not possible in such systems. Recently, IA schemes are introduced as good solutions for interference management and it is proved that IA can achieve the maximum DoF for K-user interference channel [5].

In this chapter, the preliminaries of MIMO communications such as DoF, diversity order, and STSc are explained first and then, the cooperative communications are introduced and its several protocols are also explained. And, the basic concept of IA is described.

2.1. MIMO Communications

The advantage of MIMO communications is the use of multiple antennas at both the transmitter and receiver to improve the performance of the communication systems. Since MIMO can offer significant enhancement in data throughput and reliability without additional bandwidth or increased transmit power, it has drawn great attention in wireless communications. In [11], the capacity of the Gaussian MIMO channels was derived and the advantage of MIMO communications was proved theoretically.

Consider $M \times N$ MIMO communication system, where M antennas and N antennas are equipped at the transmitter and receiver, respectively. Then, the received signal in one time slot is represented as

$$\mathbf{y} = \sqrt{\rho}\mathbf{H}\mathbf{x} + \mathbf{n} \tag{2.1}$$

where $\mathbf{H}$, $\mathbf{x}$, and $\mathbf{n}$ are the $N \times M$ channel matrix, the $M \times 1$ transmitted signal vector, and the $N \times 1$ noise vector, respectively and ρ is the value proportional to SNR. It is assumed that elements of $\mathbf{n}$ are i.i.d with $\mathcal{CN}(0, 1)$.

If the channel state is known at the transmitter and receiver, the channel capacity in (2.1) is given as [11]

$$C = \log_2 \det \left(\mathbf{I}_N + \rho \mathbf{H} \mathcal{K}_{\mathbf{x}} \mathbf{H}^\dagger \right)$$

where $\mathcal{K}_{\mathbf{x}}$ is the covariance matrix of $\mathbf{x}$.

If the channel state is not known to the transmitter, the transmitter transmits the data according to $\mathcal{K}_{\mathbf{x}} = \mathbf{I}_M / M$, which implies that the uniform power allocations and independent codes are used for each antenna. Then the channel capacity can be represented as

$$C = \sum_{m=1}^{\min(M,N)} \log_2 \left(1 + \frac{\rho}{M} \lambda_m \right) \tag{2.2}$$

where λ_m's are the eigenvalues of $\mathbf{H}\mathbf{H}^\dagger$.

Definition 2.1 (Degrees of Freedom)**.** The degrees of freedom or multiplexing gain r of the channel is defined as

$$r = \lim_{SNR \to \infty} \frac{R(SNR)}{\log(SNR)}$$

where $R(SNR)$ is an achievable rate at the SNR.

In (2.2), it can be easily seen that DoF of the MIMO channel is $\min(M, N)$.

2.2. Space-Time Coding and Selection Diversity

In this section, we review the STCs and their design criteria in order to obtain the transmit diversity together with selection diversity.

Definition 2.2 (Diversity order (gain))**.** The diversity order G_d of the system is defined as

$$G_d = - \lim_{SNR \to \infty} \frac{\log(P_e)}{\log(SNR)}$$

where P_e denotes the error probability of the system.

Comparing with Definition 2.1, the diversity order is focused on the error probability. In MIMO communications, since the reliability is also the important factor, good codes imply the one with higher DoF and diversity order.

In fact, STCs may reduce the data transmission rate to increase diversity order and thus, the STCs should satisfy the requirement of diversity order and multiplexing gain. In [1], the fundamental tradeoff of the diversity order and multiplexing gain (DMT) is analyzed as follows.

Theorem 2.3 (Diversity and multiplexing tradeoff [1])**.** Assume a code at the transmitter of a MIMO channel with M transmit antennas and N receive antennas. For a given DoF i, $i = 0, 1, \cdots, \min(M, N)$, the maximum diversity order $G_d(i)$ is given as

$$G_d(i) = (M - i)(N - i)$$

if the block length of the code is greater than or equal to $N + M - 1$.

Let $\mathbf{X}$ be a $M \times T$ space-time code matrix. Then the received signal matrix is given as

$$\mathbf{Y} = \sqrt{\rho}\mathbf{H}\mathbf{X} + \mathbf{N} \tag{2.3}$$

where $\mathbf{H} \in \mathbb{C}^{N \times M}$ and $\mathbf{N} \in \mathbb{C}^{N \times T}$ denote a channel matrix with Rayleigh fading and a AWGN matrix.

The pairwise error probability (PEP) is a useful tool to quantify the system performance of the STC. Tarokh, Seshadri, and Calderbank derived the upper bound of the PEP of STCs in (2.3) [2]. First, we consider the conditional PEP that detects $\hat{\mathbf{X}}$ while $\mathbf{X}$ is transmitted. Then, the error (difference) matrix is defined as $D(\hat{\mathbf{X}}, \mathbf{X}) = \hat{\mathbf{X}} - \mathbf{X}$. The conditional PEP is expressed as

$$\begin{aligned}
\Pr\left(\mathbf{X} \to \hat{\mathbf{X}} | \mathbf{H}\right) &= Q\left(\sqrt{\frac{\rho}{2}\|(\hat{\mathbf{X}} - \mathbf{X})\mathbf{H}\|^2}\right) \\
&= Q\left(\sqrt{\frac{\rho}{2}\sum_{n=1}^{N}\sum_{m=1}^{M}\lambda_m|\beta_{m,n}|^2}\right)
\end{aligned}$$

where γ is the received SNR and $Q(\cdot)$ is the Gaussian Q-function defined by

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp\left(-\frac{t^2}{2}\right) dt. \tag{2.4}$$

Let λ_n's be the (non-negative) eigenvalues of $A(\hat{\mathbf{X}}, \mathbf{X}) = D(\hat{\mathbf{X}}, \mathbf{X}) D(\hat{\mathbf{X}}, \mathbf{X})^\dagger$ and $\beta_{m,n}$ is the (m, n)th element of the product between the unitary matrix V and the channel matrix $\mathbf{H}$ with $A(\hat{\mathbf{X}}, \mathbf{X}) = V \Lambda V^\dagger$ for $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n)$. Using the Chernoff bound, $Q(x) \leq \frac{1}{2}\exp(-\frac{1}{2}x^2)$, the above PEP can be upper-bounded as

$$\Pr\left(\mathbf{X} \to \hat{\mathbf{X}} | \mathbf{H}\right) \leq \frac{1}{2}\exp\left(-\frac{\rho}{4}\sum_{n=1}^{N}\sum_{m=1}^{M}\lambda_m|\beta_{m,n}|^2\right).$$

Since $\beta_{n,m}$ is Gaussian, $|\beta_{m,n}|$ is Rayleigh-distributed with the PDF

$$f(|\beta_{m,n}|) = 2|\beta_{m,n}|\exp(-|\beta_{m,n}|^2).$$

Thus, the PEP can be obtained by averaging the conditional PEP over the channel matrix $\mathbf{H}$

$$\Pr\left(\mathbf{X} \to \hat{\mathbf{X}}\right) = \mathbb{E}_{\mathbf{H}}\left[\Pr\left(\mathbf{X} \to \hat{\mathbf{X}} | \mathbf{H}\right)\right]$$

$$\leq \prod_{m=1}^{M} \left(1 + \frac{\rho\lambda_m}{4}\right)^{-N}. \tag{2.5}$$

From (2.5), it can be seen that the performance of STCs depends on the rank and eigenvalues of $\hat{\mathbf{X}} - \mathbf{X}$. If its rank $r < M$, we have $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_r > 0$ and $\lambda_{r+1} = \cdots = \lambda_N = 0$. In the high SNR region, (2.5) can be upper-bounded as

$$\Pr\left(\mathbf{X} \to \hat{\mathbf{X}}\right) \leq \frac{4^{rN}}{\left(\prod_{n=1}^{r} \lambda_n\right)^N \rho^{rN}} \propto (G_c\rho)^{-G_d} \tag{2.6}$$

where G_c denote the *coding gain.*

There are two criteria to minimize the upper bound in (2.6). One is the determinant criterion to maximize the determinant of $\mathbf{A}(\hat{\mathbf{X}}, \mathbf{X})$, which corresponds to the coding gain. The other is the rank criterion to maximize the rank of $\hat{\mathbf{X}} - \mathbf{X}$, which corresponds to the diversity order. In high SNR region, the diversity order has dominant effect on the error performance.

The diversity order in Theorem 2.3 can be obtained by selection diversity techniques regardless of code length. Selection diversity techniques are based on singular value decomposition (SVD) of channel matrix, which is explained as

$$\begin{aligned}
\mathbf{Y} &= \sqrt{\rho}\mathbf{H}\mathbf{x} + \mathbf{n} \\
&= \sqrt{\rho}\mathbf{U}\Sigma\mathbf{V}^\dagger\mathbf{x} + \mathbf{n} \\
\mathbf{U}^\dagger\mathbf{Y} &= \sqrt{\rho}\mathbf{U}^\dagger\mathbf{U}\Sigma\mathbf{V}^\dagger\mathbf{V}\tilde{\mathbf{x}} + \mathbf{U}^\dagger\mathbf{n} \\
&= \sqrt{\rho}\Sigma\tilde{\mathbf{x}} + \mathbf{U}^\dagger\mathbf{n}
\end{aligned}$$

where $\mathbf{H} = \mathbf{U}\Sigma\mathbf{V}^\dagger$ by SVD and $\mathbf{x} = \mathbf{V}\tilde{\mathbf{x}}$. In this way, $\min(M, N)$ Gaussian channels can be obtained and each channel coefficient is the non-zero singular value of $\mathbf{H}$. By selecting i beamforming vectors with i largest singular values, diversity order $(M - i + 1)(N - i + 1)$ can be obtained [12].

Recently, a transmit antenna selection scheme is proposed, which is based on the maximum likelihood (ML) decoding. Since the PEP of ML decoding depends on $|\mathbf{H}_{\mathrm{sub}}\mathbf{x}|$, where $\mathbf{H}_{\mathrm{sub}}$ is a sub-matrix of $\mathbf{H}$, the subset of transmit antennas is selected to maximize $|\mathbf{H}_{\mathrm{sub}}\mathbf{x}|$. Even though the complexity of the ML decoding is much higher than that of the zero-forcing, this selection scheme with ML decoding can achieve diversity order $(M - i + 1)N$ [13].

2.3. Cooperative Communications

Even though MIMO systems have advantage from the viewpoints of throughput and reliability, virtual MIMO systems are required for the next generation wireless communication systems due to the difficulty of the implementation of the multiple antennas in a mobile device. The virtual MIMO systems are implemented by the cooperative communication networks using the relays. In general, cooperative communication networks consist of source (transmitter), relay, and destination (receiver). Relay nodes are introduced to help the communication between the source and destination. Since Cover and El Gamal [14] derived some capacity bound on several relay channels, many researchers have studied the relay channels theoretically and practically.

In fact, many schemes of cooperative communication networks are focused on the increase of the diversity order. The distributed STC (DSTC) is a well-known cooperative diversity technique, which combines the signals from the source and relays and constructs STC.

In general, the protocols of cooperative communications are classified according to the operation of relay. The first protocol is AF protocol, where the relay amplifies the received signal and forwards it to the destination. The second one is DF protocol, where the relay decodes the received signal and reencodes and forwards it to the destination. Also, the protocols of cooperative communications are classified into OT and NT protocols, where the source and relay transmit signal separately or simultaneously.

For more details, we look into the input-output relation of the protocols of cooperative communications. Fig. 2.1 describes the AF protocol. The signals transmitted from S (source node) during the first and second time slots are denoted as x_1 and x_2. The data symbols may be chosen from a complex-valued finite constellation such as QAM or from some codebooks. And then, the signals received at R (relay node) and D (destination node) in the first time slot are given as

$$y_{\mathsf{R}} = \sqrt{P_{\mathsf{SR}}} h_{\mathsf{SR}} x_1 + n_{\mathsf{R}}$$
$$y_{\mathsf{D1}} = \sqrt{P_{\mathsf{SD}}} h_{\mathsf{SD}} x_1 + n_{\mathsf{D1}}$$

where P_{SR} and P_{SD} are the average signal energy received at R and D

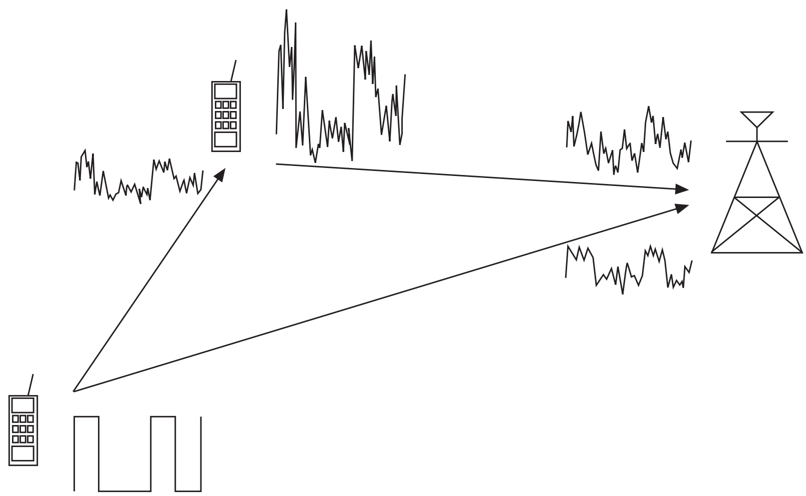

Figure 2.1: Amplify-and-forward protocol [15].

over one symbol period through the link (having accounted for path loss and shadowing between S and D), h_{SR} and h_{SD} are the random, complex-valued, unit-power channel gain, i.e., $h_{\mathsf{SR}},\ h_{\mathsf{SD}} \sim \mathcal{CN}(0,1)$ between S and D, and $n_{\mathsf{R}} \sim \mathcal{CN}(0,1)$ and $n_{\mathsf{D}1} \sim \mathcal{CN}(0,1)$ are AWGN, respectively. Note that $P_{\mathsf{SR}} \neq P_{\mathsf{SD}}$ generally due to differences in path loss and shadowing between $\mathsf{S} \to \mathsf{R}$ and $\mathsf{S} \to \mathsf{D}$ links.

As an intermediate step for orthogonal AF (OAF), R normalizes the received signal by a factor of $\sqrt{\mathbb{E}\left[|y_{\mathsf{R}}|^2\right]}$ (so that the average energy is unity) and retransmits the signal during the second time slot. D receives the signal transmitted from R during the second time slot according to

$$\begin{aligned} y_{\mathsf{D}2} &= \sqrt{P_{\mathsf{RD}}} h_{\mathsf{RD}} \frac{y_{\mathsf{R}}}{\sqrt{\mathbb{E}\left[|y_{\mathsf{R}}|^2\right]}} + n_{\mathsf{D}2} \\ &= \sqrt{\frac{P_{\mathsf{SR}} P_{\mathsf{RD}}}{P_{\mathsf{SR}} + 1}} h_{\mathsf{SR}} h_{\mathsf{RD}} x_1 + n_{\mathsf{D}} \end{aligned}$$

where the effective noise in the second time slot n_{D} is represented as

$$n_{\mathsf{D}} = n_{\mathsf{D}2} + \sqrt{\frac{P_{\mathsf{RD}}}{P_{\mathsf{SR}}+1}}h_{\mathsf{RD}}n_{\mathsf{R}}.$$

For NAF protocol, since the source and the relay transmit their signal simultaneously in the second time slot, the received signal at the destination is given as

$$y_{\mathsf{D}2} = \sqrt{\frac{P_{\mathsf{SR}}P_{\mathsf{RD}}}{P_{\mathsf{SR}}+1}}h_{\mathsf{SR}}h_{\mathsf{RD}}x_1 + h_{\mathsf{SD}}x_2 + n_{\mathsf{D}}.$$

Therefore, the effective input-output relations for OAF protocol can be expressed as

$$\underbrace{\begin{bmatrix} y_{\mathsf{D}1} \\ y_{\mathsf{D}2} \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} \sqrt{P_{\mathsf{SD}}}h_{\mathsf{SD}} \\ \sqrt{\frac{P_{\mathsf{SR}}P_{\mathsf{RD}}}{P_{\mathsf{SR}}+1}}h_{\mathsf{SR}}h_{\mathsf{RD}} \end{bmatrix}}_{\mathbf{h}} x_1 + \underbrace{\begin{bmatrix} n_{\mathsf{D}1} \\ n_{\mathsf{D}} \end{bmatrix}}_{\mathbf{n}}$$

and for NAF protocol, we have

$$\underbrace{\begin{bmatrix} y_{\mathsf{D}1} \\ y_{\mathsf{D}2} \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} \sqrt{P_{\mathsf{SD}}}h_{\mathsf{SD}} & 0 \\ \sqrt{\frac{P_{\mathsf{SR}}P_{\mathsf{RD}}}{P_{\mathsf{SR}}+1}}h_{\mathsf{SR}}h_{\mathsf{RD}} & \sqrt{P_{\mathsf{SD}}}h_{\mathsf{SD}} \end{bmatrix}}_{\mathbf{h}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} n_{\mathsf{D}1} \\ n_{\mathsf{D}} \end{bmatrix}}_{\mathbf{n}},$$

respectively.

In DF (NDF) protocol, the signals received at R and D during the first time slot are identical to those for the AF protocol. Unlike AF protocol, R demodulates and decodes the signal received during the first time slot. Assuming that the signal is decoded correctly and retransmitted, we obtain

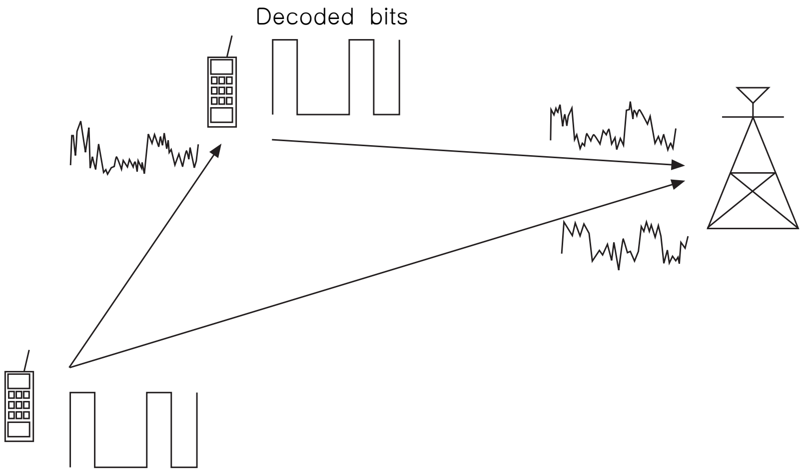

Figure 2.2: Decode-and-forward protocol [15].

the following relation for ODF protocol

$$y_{\mathsf{D2}} = \sqrt{P_{\mathsf{RD}}} h_{\mathsf{RD}} x + n_{\mathsf{D2}}.$$

This can be done with cyclic redundancy check (CRC) with high order polynomial and automatic repeat request (ARQ).

And then, the effective input-output relation in the ODF protocol can be summarized as

$$\underbrace{\begin{bmatrix} y_{\mathsf{D1}} \\ y_{\mathsf{D2}} \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} \sqrt{P_{\mathsf{SD}}} h_{\mathsf{SD}} \\ \sqrt{P_{\mathsf{RD}}} h_{\mathsf{RD}} \end{bmatrix}}_{\mathbf{h}} x_1 + \underbrace{\begin{bmatrix} n_{\mathsf{D1}} \\ n_{\mathsf{D2}} \end{bmatrix}}_{\mathbf{n}}.$$

Similar to NAF protocol, NDF protocol can be easily seen that the effective input-output relation is given as

$$\underbrace{\begin{bmatrix} y_{\mathsf{D}1} \\ y_{\mathsf{D}2} \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} \sqrt{P_{\mathsf{SD}}} h_{\mathsf{SD}} & 0 \\ \sqrt{P_{\mathsf{RD}}} h_{\mathsf{RD}} & \sqrt{P_{\mathsf{SD}}} h_{\mathsf{SD}} \end{bmatrix}}_{\mathbf{h}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} n_{\mathsf{D}1} \\ n_{\mathsf{D}2} \end{bmatrix}}_{\mathbf{n}}.$$

In fact, since CRC and ARQ reduce the throughput greatly and there can also exist the error for CRC and ARQ, it is not assumed that the signal is always decoded correctly at the relay in the recent trend. Therefore, the relay may transmit error symbol and it causes the error propagation. In order to resolve this problem, the decoding at the receiver is done considering error probability at the relay.

2.4. Interference Alignment

In order to satisfy the requirement of wireless communication service, such as extremely high data rate and reliability, MIMO technology becomes common and some small cellular systems are considered such as pico- and femto-cells. Especially, by the heterogeneous networks, as cell size becomes smaller than before, the spectral efficiency and power consumption can be solved partly improved.

However, as the cell size becomes smaller, the cell boundary region becomes wider. This implies that the interference has the bad effect on the reliable communications specially in the cell boundary region. Therefore, the interference management becomes a hot issue of the wireless communications.

Conventionally, there are three interference management approaches:

1) Decode:

- Interference is strong.
- The interference signal can be decoded and then subtracted from the desired signal.

2) Treat as noise:

- Interference is weak.
- The interference signal is treated as noise and single user encoding/decoding suffices.

3) Orthogonalization:

- The strength of interference is comparable to the desired signal.
- This approach is to orthogonalize interferences and desired signals in time, frequency, or code.

When interference signal power is comparable to the desired signal, approaches on 'decode' and 'treat as noise' do not guarantee high reliability and 'orthogonalization' reduces the throughput. In this situation, IA scheme is drawing great attention as a solution for the above conventional approaches. The basic idea of IA is separating the space of interference signal from the spaces of desired signal. And by maximizing the overlap between the spaces of all interferences at each receiver, it increases the DoF of the desired signal.

Fig. 2.3 describes the basic concept of IA. In Fig. 2.3, four interference signals align to the two-dimensional subspace and thus, by using $\mathbf{u}$, the

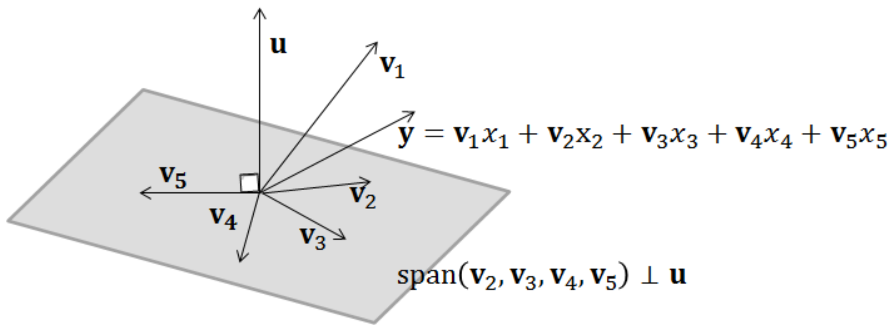

Figure 2.3: Graphical interpretation on IA.

desired signal $\mathbf{v}_1$ can be extracted without interference signal. In more details, $\mathbf{u}$ is orthogonal to the $\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5$ and thus

$$\begin{aligned}\mathbf{u}^\dagger \mathbf{y} &= \mathbf{u}^\dagger \mathbf{v}_1 x_1 + \mathbf{u}^\dagger \mathbf{v}_2 x_2 + \mathbf{u}^\dagger \mathbf{v}_3 x_3 + \mathbf{u}^\dagger \mathbf{v}_4 x_4 + \mathbf{u}^\dagger \mathbf{v}_5 x_5 \\ &= \mathbf{u}^\dagger \mathbf{v}_1 x_1.\end{aligned}$$

In this case, orthogonalization can provide 1/5 DoF but IA achieves 1/3 DoF.

Usually, IA is implemented by using multiple antennas, carriers, or time extension. Cadambe and Jafar [5, 6] proposed IA scheme for K-user interference channel and X channel by using time extension. However, it is assumed that the channel is time-varying and the global CSI is available at the transmitters, which implies that each transmitter knows the CSIs of all links between transmitters and receivers. These assumptions are infeasible because for global CSI, each transmitter requires lots of feedback information and the channel state may vary during feedback of global CSI. Therefore, IA under the quasi-static channel is recognized more practical and Yetis et. al. [17] derived feasibility condition for IA to use multiple antennas.

In general, IA is closely connected with MIMO technology because many IA schemes are implemented by using multiple antennas. As the number of antennas increases at the node, the dimensions of transmit and received signal spaces also increase. Therefore, if the transmitter or receiver have plenty of dimension of signal spaces, the perfect IA can be implemented easily. However, the numbers of antennas for IA are restricted from the size of device. This implies that the virtual antenna system is also recommended for IA and thus, recently, some relay-aided IA schemes are introduced [7, 8, 9], where relays are operated in AF-mode. Since these relay-aided IA schemes are implemented under the quasi-static channel, they can be good alternatives for the conventional IA.

Chapter 3. Relay On-Off Threshold for NDF Protocol with Distributed Orthogonal Space-Time Block Codes

3.1. Introduction

Attenuation and fading in the multipath wireless environment make it difficult for a receiver to correctly decode a received signal. Therefore, replicas of a transmitted signal are sent to the receiver to improve the detection performance.

Tarokh *et al.* [2] proposed STCs to achieve transmit diversity and their design criteria. Alamouti code was proposed in [3], which was extended to OSTBCs in [18]. Also, the symbol error rate (SER) and bit error rate (BER) of OSTBCs with QAM were derived in [19] and [20], respectively.

The cooperative diversity which is obtained by utilizing relays between source and destination has been actively studied since Cover and El Gamal's work [14]. Laneman *et al.* [16] developed some cooperative diversity schemes based on repetition codes and analyzed their performance, and Wang *et al.* [21] developed a good demodulation scheme for the DF protocol proposed in [16].

Recently, various results on how to select the best relay among multiple relays have been reported. Bletsas *et al.* [22] proposed an opportunistic relaying which selects a relay only by using local channel information. Jing and Jafarkhani [23] proposed a multiple-relay selection scheme for AF protocol and Yi and Kim [24] analyzed the diversity order of the DF protocol with relay selection.

On the other hand, a relay on-off scheme for the DF protocol according to the channel state was proposed in [16]. Recently, another relay selection scheme based on the ROT for the DF protocol was also proposed in [25]. In both schemes, if the received SNR at the relay is larger than the ROT, the relay transmits a signal and otherwise, the relay does not transmit a signal. However, the optimal ROT was not derived analytically.

For ODF protocol and binary phase shift keying (BPSK), Siriwongpairat et al. [26] derived the ROT over the Rayleigh fading channel and Ikki and Ahamed [27] derived the ROT over the Nakagami-m fading channel. However, until now, for M-QAM and NDF protocol in which a source also transmits a signal when a relay transmits a signal, the ROT has not been investigated.

It is well known that SAS can gain additional diversity for the NDF protocol by using different antennas at the source in the first and second phases [10] and thus we consider SAS in this chapter.

The ML decoding is impractical because its computational complexity increases exponentially with the signal constellation size. Therefore, we will use the low-complexity decoding scheme for OSTBCs given in [28, 29], which is called LC decoding.

In this chapter, an ROT is derived for the NDF protocol with dis-

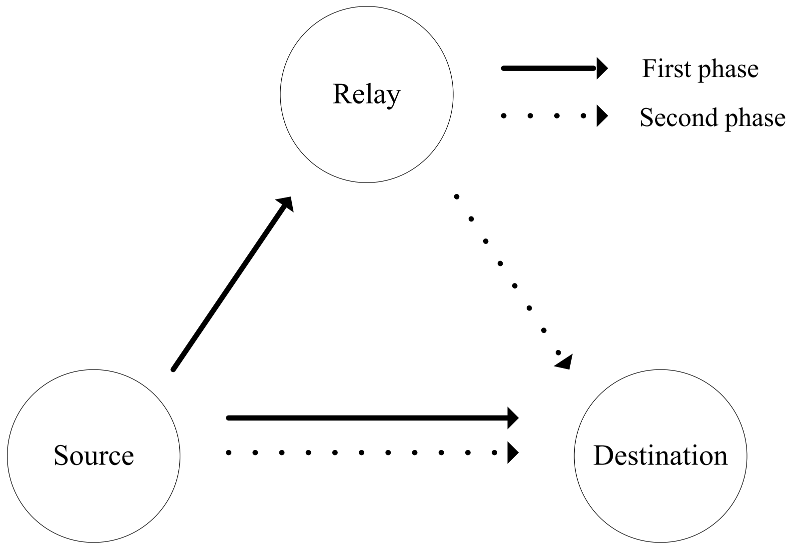

Figure 3.1: An NDP protocol for three-terminal cooperative communication network.

tributed OSTBCs (DOSTBCs) and M-QAM such that the SER of LC decoding is minimized over the Rayleigh fading channel. Until now, this kind of ROT has not been investigated yet. And, the diversity order of the proposed relay on-off scheme is also derived.

3.2. System Models and Linear Combining Decoding

For the three-terminal cooperative communication network in Fig. 3.1, it is assumed that source (S), relay (R), and destination (D) can have multiple antennas and the channels are quasi-static Rayleigh fading ones. Since NT protocol is assumed, both source and relay transmit signals in the second phase. It is also assumed that the destination has the perfect

CSI of S-D and R-D channels and the relay knows the CSI of S-R channel and the mean and variance of S-D and R-D channel coefficients. For simplicity, Gray-mapped M-QAM is assumed.

If the SNR at the relay is larger than an ROT T in the DF protocol, the relay transmits a signal and otherwise, the relay does not transmit a signal. Note that it is assumed that the source and destination know whether the relay is on or off. In the cooperative communication network, if the source uses the same antenna in the first and second phases, the channel coefficients for these two phases become identical. Therefore, in order to increase the diversity order, it is assumed that the SAS in [10] is used.

3.2.1. System Models of NDF Protocol

It is assumed that the numbers of antennas of the source, relay, and destination are M_{S}, M_{R}, and M_{D}, respectively. Let $\mathbf{x}$ be the message vector consisting of L independent symbols. $\mathbf{C}_{S,1}(\mathbf{x}) \in \mathcal{C}^{\mathcal{T} \times M_{\mathsf{S}}}$ denotes the codeword of OSTBC transmitted from the source, where $\mathcal{T}$ denotes the number of transmissions at each source antenna. In the first phase, the received signal matrices $\mathbf{Y}_{\mathsf{R}}$ and $\mathbf{Y}_{\mathsf{D}1}$ at the relay and destination can be written as

$$\mathbf{Y}_{\mathsf{R}} = \sqrt{p_1 \rho}\mathbf{C}_{\mathsf{S},1}(\mathbf{x})\mathbf{H} + \mathbf{N}_{\mathsf{R}} \tag{3.1}$$

$$\mathbf{Y}_{\mathsf{D}1} = \sqrt{p_1 \rho}\mathbf{C}_{\mathsf{S},1}(\mathbf{x})\mathbf{G}_1 + \mathbf{N}_{\mathsf{D},1} \tag{3.2}$$

where $\mathbf{H} \in \mathcal{C}^{M_{\mathsf{S}} \times M_{\mathsf{R}}}$ and $\mathbf{G}_1 \in \mathcal{C}^{M_{\mathsf{S}} \times M_{\mathsf{D}}}$ are the S-R and S-D channel matrices, and $\mathbf{N}_{\mathsf{R}} \in \mathcal{C}^{\mathcal{T} \times M_{\mathsf{R}}}$ and $\mathbf{N}_{\mathsf{D},1} \in \mathcal{C}^{\mathcal{T} \times M_{\mathsf{D}}}$ are the noise matrices at the relay and destination. Note that $p_1\rho$ is the power of the signal from the source and ρ is a parameter linearly proportional to the average transmit SNR.

In the second phase, let $\mathbf{x}^R$ denote the decoded message vector at the relay and $\mathbf{C}_{\mathsf{S},2}(\mathbf{x}) \in \mathcal{C}^{\mathcal{T} \times M_{\mathsf{S}}}$ and $\mathbf{C}_{\mathsf{R},2}(\mathbf{x}^{\mathsf{R}}) \in \mathcal{C}^{\mathcal{T} \times M_{\mathsf{R}}}$ denote the codewords of OSTBCs corresponding to $\mathbf{x}$ and $\mathbf{x}^{\mathsf{R}}$ transmitted from the source and relay, respectively. When the relay is on, the received signal at the destination in the second phase is given as

$$\mathbf{Y}_{\mathsf{D}2} = \begin{bmatrix} \sqrt{p_2\rho}\mathbf{C}_{\mathsf{S},2}(\mathbf{x}) & \sqrt{p_3\rho}\mathbf{C}_{\mathsf{R},2}(\mathbf{x}^{\mathsf{R}}) \end{bmatrix} \begin{bmatrix} \mathbf{G}_2 \\ \mathbf{F} \end{bmatrix} + \mathbf{N}_{\mathsf{D},2} \tag{3.3}$$

where $\mathbf{G}_2 \in \mathcal{C}^{M_{\mathsf{S}} \times M_{\mathsf{D}}}$ and $\mathbf{F} \in \mathcal{C}^{M_{\mathsf{R}} \times M_{\mathsf{D}}}$ are the S-D and R-D channel matrices, and $\mathbf{N}_{\mathsf{D},2} \in \mathcal{C}^{\mathcal{T} \times M_{\mathsf{D}}}$ is the noise matrix at the destination. Note that $p_2\rho$ and $p_3\rho$ denote the signal powers from the source and relay in the second phase, respectively. Then $\begin{bmatrix} \mathbf{C}_{\mathsf{S},2}(\mathbf{x}) & \mathbf{C}_{\mathsf{R},2}(\mathbf{x}^{\mathsf{R}}) \end{bmatrix}$ also forms an OSTBC if $\mathbf{x}^{\mathsf{R}} = \mathbf{x}$. The entries of $\mathbf{H}$, $\mathbf{G}_1$, $\mathbf{G}_2$, and $\mathbf{F}$ are independently distributed as $CN(0, \sigma^2_{\mathsf{SR}})$, $CN(0, \sigma^2_{\mathsf{SD}})$, $CN(0, \sigma^2_{\mathsf{SD}})$, and $CN(0, \sigma^2_{\mathsf{RD}})$, respectively. The entries of $\mathbf{N}_{\mathsf{R}}$, $\mathbf{N}_{\mathsf{D},1}$, and $\mathbf{N}_{\mathsf{D},2}$ are *i.i.d.* random variables distributed as $CN(0, 1)$.

When the relay is off, the received signal at the destination in the second phase is given as

$$\mathbf{Y}_{\mathsf{D}2} = \sqrt{p_2\rho}\mathbf{C}_{\mathsf{S},2}(\mathbf{x})\mathbf{G}_2 + \mathbf{N}_{\mathsf{D},2}. \tag{3.4}$$

Therefore, by using (3.2), (3.3), and (3.4), the received signal matrix can be expressed as

$$\mathbf{Y}_{\mathsf{D}} = \begin{bmatrix} \mathbf{Y}_{\mathsf{D}1} \\ \mathbf{Y}_{\mathsf{D}2} \end{bmatrix}.$$

3.2.2. Linear Combining Decoding

The optimal ML decoding at the destination is done by selecting $\hat{\mathbf{x}}$ such as

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} \log \Pr(\mathbf{Y}_{\mathsf{D}}|\mathbf{x}) = \arg\max_{\mathbf{x}} \log \sum_{\mathbf{x}_{\mathsf{R}}} \Pr(\mathbf{x}_{\mathsf{R}}|\mathbf{x}) \Pr(\mathbf{Y}_{\mathsf{D}}|\mathbf{x}_{\mathsf{R}}, \mathbf{x}).$$

Since the complexity of ML decoding increases exponentially to the constellation size and the analysis of ML decoding is very difficult, the simple and practical LC decoding [28, 29] will be considered in this chapter. It is clear that LC decoding is equivalent to ML decoding for OSTBCs but it is not true for DOSTBCs because the relay may transmit erroneously decoded symbols.

For the cooperative communication network, the LC decoding operates twice. First, the LC decoding is done for $\mathbf{Y}_{\mathsf{D}1}$ and $\mathbf{Y}_{\mathsf{D}2}$, respectively, before the decision on $\mathbf{x}$ is made at the destination. Then, the destination combines the decoder outputs for $\mathbf{Y}_{\mathsf{D}1}$ and $\mathbf{Y}_{\mathsf{D}2}$. Finally, the decision is made for $\mathbf{x}$ by using this combined output.

3.3. Relay On-Off Threshold and Diversity Analysis

If the S-R channel or R-D channel is not good enough for the reliable communication, a relay cannot help the destination that much. Therefore, it is important to decide whether a relay should be used or not depending on the states of S-R and R-D channels. In general, a relay is used in the DF protocol only when the received SNR at the relay is greater than the ROT. If the relay correctly decodes the information, it can be thought as a virtual antenna for the source to destination and the diversity gain can be obtained even though the R-D channel is not good. Therefore, an ROT can be determined only by monitoring S-R channel state [16, 26, 27].

If the ROT is too low, the relay may transmit many erroneous data under the bad S-R channel condition, which causes many decoding errors at the destination. On the contrary, if the ROT is too high, the relay is rarely used and the cooperative diversity cannot be achieved. Therefore, the optimal ROT for the NDF protocol with DOSTBCs and M-QAM is derived in this section.

It can be assumed that, in high SNR region, only one symbol error occurs among L symbols in a codeword and further a symbol error is caused by one-bit error. Let $u = \sum_{i=1}^{M_\mathsf{S}} \sum_{j=1}^{M_\mathsf{R}} |h_{i,j}|^2$, $g_1 = \sum_{i=1}^{M_\mathsf{S}} \sum_{j=1}^{M_\mathsf{D}} |g_{1,i,j}|^2$, $g_2 = \sum_{i=1}^{M_\mathsf{S}} \sum_{j=1}^{M_\mathsf{D}} |g_{2,i,j}|^2$, and $w = \sum_{i=1}^{M_\mathsf{D}} \sum_{j=1}^{M_\mathsf{R}} |f_{i,j}|^2$, where $g_{k,i,j}$ denotes the channel coefficient between the ith source antenna and the jth destination antenna in the kth phase, and $h_{i,j}$ denotes the channel coefficient between the ith source antenna and the jth relay antenna, and $f_{i,j}$ denotes the channel coefficient between the ith relay antenna and the jth destination antenna. Clearly, u, g_1, g_2, and w are Erlang distributed [31].

3.3.1. Relay On-Off Threshold

To derive the ROT minimizing the SER, the SER should be derived first. For the NDF protocol using a relay on-off scheme, the symbol error event at the destination can be divided into three cases. The first case is that symbol error occurs at the destination when the relay is off. The second case is that symbol error does not occur at the relay but symbol error occurs at the destination when the relay is on. The third case is that symbol error occurs at both the relay and destination when the relay is on. Therefore, the SER $P_e(T)$ with the ROT T at the destination is expressed as

$$\begin{aligned}
&P_e(T) \\
&= \underbrace{\Pr(\text{symbol error } |\mathsf{R} \text{ off})\Pr(\mathsf{R} \text{ off})}_{\text{Case 1}} \\
&+ \underbrace{\Pr(\text{symbol error } | \ \mathsf{R} \text{ on, no symbol error at } \mathsf{R})\Pr(\mathsf{R} \text{ on, no symbol error at } \mathsf{R})}_{\text{Case 2}} \\
&+ \underbrace{\Pr(\text{symbol error } | \ \mathsf{R} \text{ on, symbol error at } \mathsf{R})\Pr(\text{relay on, symbol error at the } \mathsf{R})}_{\text{Case 3}}.
\end{aligned} \tag{3.5}$$

As SNR increases, the probability of Case 3 in (3.5) can be approximated as

$$\underbrace{\Pr(\text{symbol error} \mid \mathsf{R}\text{ on, symbol error at }\mathsf{R})\Pr(\mathsf{R}\text{ on, symbol error at the }\mathsf{R})}_{\text{Case 3}}$$
$$\approx \underbrace{\Pr(\text{symbol error} \mid \mathsf{R}\text{ on, one bit error at }\mathsf{R})\Pr(\mathsf{R}\text{ on, one bit error at the }\mathsf{R})}_{\text{Case 3}}. \tag{3.6}$$

The PDF of Erlang random variable u is given as

$$f(u) = \frac{\lambda e^{-\lambda u}(\lambda u)^{M_{\mathsf{S}}M_{\mathsf{R}}-1}}{(M_{\mathsf{S}}M_{\mathsf{R}}-1)!} \tag{3.7}$$

where $\lambda = 1/\sigma_{\mathsf{SR}}^2$. Let P_{e1} be the SER at the destination when the relay is off, P_{e2} be the SER at the destination when the relay is on and no decoding error occurs at the relay, and P_{e3} be the SER at the destination when the relay is on and the decoding error occurs at the relay. Using (3.5) and (3.6), the SER at the destination is derived as

$$\begin{aligned}
&P_e(T) \\
&= P_{e1}\int_0^{\frac{T}{p_1\rho}} f(u)du + P_{e2}\int_{\frac{T}{p_1\rho}}^{\infty} \underbrace{\left\{1-4\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3p_1\rho u}{M-1}}\right)\right\}^L}_{\text{probability that no symbol error occurs at }\mathsf{R}} f(u)du \\
&+ P_{e3}\int_{\frac{T}{p_1\rho}}^{\infty} \underbrace{\left[1-\left\{1-4\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3p_1\rho u}{M-1}}\right)\right\}^L\right]}_{\text{probability that symbol errors occur at }\mathsf{R}} f(u)du \\
&\approx P_{e1}\int_0^{\frac{T}{p_1\rho}} f(u)du + P_{e2}\int_{\frac{T}{p_1\rho}}^{\infty} \left\{1-4L\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3p_1\rho u}{M-1}}\right)\right\} f(u)du
\end{aligned}$$

$$+ P_{e3} \int_{\frac{T}{p_1 \rho}}^{\infty} \underbrace{4L \left(1 - \frac{1}{\sqrt{M}}\right) Q \left(\sqrt{\frac{3 p_1 \rho u}{M-1}}\right)}_{\approx \text{ one bit error probability at } \mathsf{R}} f(u) du \tag{3.8}$$

where P_{e3} can be approximated as the symbol error probability at the destination when one bit error causes one symbol error at the relay. Then, by solving $\frac{dP_e(T)}{dT} = 0$ for (3.8), the ROT to minimize (3.8) is given as

$$T = \begin{cases} \frac{M-1}{3} \left[Q^{-1} \left(\frac{P_{e2} - P_{e1}}{4L\left(1 - \frac{1}{\sqrt{M}}\right)(P_{e2} - P_{e3})} \right) \right]^2, & \text{for } P_{e1} > P_{e2} \\ \infty, & \text{otherwise.} \end{cases} \tag{3.9}$$

Since some approximations are used to derive the ROT, it is clearly sub-optimal. However, through simulation, it will be shown that this ROT approaches the optimal ROT as SNR increases.

3.3.2. Relay On-Off Threshold for LC Decoding

In this subsection, we derive P_{e1}, P_{e2}, and P_{e3} for LC decoding and obtain the ROT based on them.

Case 1) Relay is off:

Since only the source transmits signals, P_{e1} can be written as [19]

$$P_{e1} \approx 4 \left(1 - \frac{1}{\sqrt{M}}\right) E \left[Q \left(d \sqrt{2\rho \left(p_1 g_1 + p_2 g_2\right)} \right) \right] \tag{3.10}$$

where d denotes the distance between one symbol point and the adjacent decision boundary in the rectangular QAM and the closed form of P_{e1} can be derived by the result in [30].

Case 2) Relay is on and no decoding error occurs at the relay:

Since the relay transmits correct data, P_{e2} can be written as [19]

$$P_{e2} \approx 4\left(1-\frac{1}{\sqrt{M}}\right) E\left[Q\left(d\sqrt{2\rho(p_1 g_1 + p_2 g_2 + p_3 w)}\right)\right] \tag{3.11}$$

and its closed form can be derived by the result in [30].

Case 3) Relay is on and the decoding error occurs at the relay:

Since Gray-mapped M-QAM is assumed, the SER is dominantly determined by the single-bit error at the relay as SNR increases. Also, the most frequent symbol error event at the relay is one symbol error among L symbols. Therefore, P_{e3} can be approximated as the SER at the destination when one-bit error occurs at the relay.

Since the SER depends on the LC decoder output before the decision is made, we confirm how the one bit error at the relay affects the LC decoder output at the destination and then derive the SER.

When the relay erroneously decodes x_k to $x_k + 2d$, the LC decoder output s_k for x_k at the destination can be written as

$$s_{k,R} = \mathcal{R}(s_k) = \sqrt{\rho}(p_1 g_1 + p_2 g_2 + p_3 w)\mathcal{R}(x_k) + 2dp_3 w + \mathcal{R}(n_k) \tag{3.12}$$

$$s_{k,I} = \mathcal{I}(s_k) = \sqrt{\rho}\,(p_1 g_1 + p_2 g_2 + p_3 w)\,\mathcal{I}(x_k) + \mathcal{I}(n_k). \tag{3.13}$$

Next, let us investigate the LC decoder output $s_i, i \neq k$ for the x_i which is not erroneously decoded at the relay. For OSTBCs, when the LC decoding is performed for the data symbol x_i, the terms related to the other symbols are canceled out and thus the decoder output at the destination becomes equivalent to (3.13). However, for DOSTBCs, if one bit error for x_k occurs at the relay, the terms related to x_k possibly affect the decoder output for the other symbol at the destination. Therefore, the decoder output for x_i at the destination can be divided into two cases. First, the terms related to x_k do not affect s_i such as

$$\begin{aligned} s_{i,R} &= \mathcal{R}(s_i) = \sqrt{\rho}(p_1 g_1 + p_2 g_2 + p_3 w)\mathcal{R}(x_i) + \mathcal{R}(n_i) \\ s_{i,I} &= \mathcal{I}(s_i) = \sqrt{\rho}(p_1 g_1 + p_2 g_2 + p_3 w)\mathcal{I}(x_i) + \mathcal{I}(n_i). \end{aligned} \tag{3.14}$$

Second, the terms related to x_k affect s_i due to one bit error for x_k at the relay such as

$$\begin{aligned} s_{i,R} = \sqrt{\rho}\big\{&(p_1 g_1 + p_2 g_2 + p_3 w)\mathcal{R}(x_i) \\ &+ \sum_{(i,j),(l,k)\in D} 2d\sqrt{\rho p_2 p_3}\mathcal{R}(g_{2,i,j} \circ f_{l,k})\big\} + \mathcal{R}(n_i) \\ s_{i,I} = \sqrt{\rho}\big\{&(p_1 g_1 + p_2 g_2 + p_3 w)\mathcal{I}(x_i) \\ &+ \sum_{(i,j),(l,k)\in D} 2d\sqrt{\rho p_2 p_3}\mathcal{I}(g_{2,i,j} \circ f_{l,k})\big\} + \mathcal{I}(n_i) \end{aligned} \tag{3.15}$$

where n_i is distributed as $CN(0, p_1 g_1 + p_2 g_2 + p_3 w)$, $g_{2,i,j} \circ f_{l,k}$ denotes either $g_{2,i,j} f_{l,k}^*$ or $g_{2,i,j}^* f_{l,k}$ depending on the used DOSTBC, and D is the set of indices of channel coefficients that are not canceled out due to one bit error for x_k.

As an example, consider a DOSTBC using the following OSTBCs for $M_S = 2$ and $M_R = 2$

$$\mathbf{C}_{\mathsf{S},1}(\mathbf{x}) = \begin{bmatrix} x_1 & -x_2^* & x_3^* & 0 \\ x_2 & x_1^* & 0 & x_3^* \end{bmatrix}^T, \mathbf{C}_{\mathsf{S},2}(\mathbf{x}) = \begin{bmatrix} x_1 & -x_2^* & x_3^* & 0 \\ x_2 & x_1^* & 0 & x_3^* \end{bmatrix}^T$$

$$\mathbf{C}_{\mathsf{R},2}(\mathbf{x}) = \begin{bmatrix} x_3 & 0 & -x_1^* & -x_2^* \\ 0 & x_3 & x_2 & -x_1 \end{bmatrix}^T . \tag{3.16}$$

If an error occurs for x_1 at the relay, $s_{2,R}$ and $s_{2,I}$ are given as (3.14) and $s_{3,R}$ and $s_{3,I}$ are given as (3.15). If an error occurs for x_2 at the relay, $s_{1,R}$ and $s_{1,I}$ are given as (3.14) and $s_{3,R}$ and $s_{3,I}$ are given as (3.15). If an error occurs for x_3 at the relay, $s_{1,R}$, $s_{1,I}$, $s_{2,R}$, and $s_{2,I}$ are given as (3.15).

From the LC decoder outputs (3.12), (3.13), (3.14), and (3.15), we can check the error cases at the destination and then calculate P_{e3}.

- The SER for x_k (the error symbol at the relay) at the destination:

  First, we consider the case that the error occurs at the destination when decoding s_k in (3.12) and (3.13). Note that k is the index of the error symbol at the relay.

  - The error case that the destination decodes $\mathcal{R}(x_k)$ to $\mathcal{R}(x_k) + 2d$;

    From (3.12), if $s_{k,R} > \sqrt{\rho}\,\{p_1 g_1 + p_2 g_2 + p_3 w\}\ (\mathcal{R}(x_k) + d)$ and there is an adjacent symbol of x_k in this direction, the destination can erroneously decode $\mathcal{R}(x_k)$ to $\mathcal{R}(x_k) + 2d$ with the following SER

    $$P_{e3,1} = E\left[Q\left(\frac{d\sqrt{2\rho}\,(p_1 g_1 + p_2 g_2 - p_3 w)}{\sqrt{p_1 g_1 + p_2 g_2 + p_3 w}}\right)\right]. \tag{3.17}$$

    Note that this symbol error at the destination is the same as the symbol error at the relay.

– The error case that the destination decodes $\mathcal{R}(x_k)$ to $\mathcal{R}(x_k) - 2d$;

From (3.12), if $s_{k,R} < \sqrt{\rho}\,\{p_1 g_1 + p_2 g_2 + p_3 w\}\,(\mathcal{R}(x_k) - d)$, this symbol error for $\mathcal{R}(x_k)$ can also occur with the following SER

$$P'_{e3,1} = E\left[Q\left(\frac{d\sqrt{2\rho}(p_1 g_1 + p_2 g_2 + 3p_3 w)}{\sqrt{p_1 g_1 + p_2 g_2 + p_3 w}}\right)\right].$$

– The error case that the destination decodes $\mathcal{I}(x_k)$ to $\mathcal{I}(x_k) \pm 2d$;

From (3.13), if $s_{k,I} > \sqrt{\rho}\,\{p_1 g_1 + p_2 g_2 + p_3 w\}\,(\mathcal{I}(x_k) + d)$ or $s_{k,I} < \sqrt{\rho}\{p_1 g_1 + p_2 g_2 + p_3 w\}(\mathcal{I}(x_k) - d)$, a symbol error for $\mathcal{I}(x_k)$ can occur at the destination with the following SER

$$P_{e3,2} = E\left[Q\left(d\sqrt{2\rho\,(p_1 g_1 + p_2 g_2 + p_3 w)}\right)\right].$$

Note that the relay can erroneously decodes x_k to $x_k - 2d$ or $x_k \pm j2d$. However, the identical LC decoder outputs to (3.12) and (3.13) are obtained because the rectangular QAM is considered. Thus, for those cases, we can also obtain $P_{e3,1}, P'_{e3,1}$, and $P_{e3,2}$, similarly. Therefore, when one bit error occurs for x_k at the relay, the SER for x_k at the destination is given as the linear combination of $P_{e3,1}, P'_{e3,1}$, and $P_{e3,2}$, that is,

$$P_{error,k} = P_{e3,1} + \beta P'_{e3,1} + \gamma_k P_{e3,2} \tag{3.18}$$

where β and γ_k are constants determined by the constellation and structure of DOSTBCs. The coefficient of $P_{e3,1}$ in (3.18) is 1 because

$P_{e3,1}$ can be regarded as the probability that the same error at the relay also occurs at the destination.

- The SER for $x_i, i \neq k$ (no error symbol at the relay):

  The SER for x_i can be obtained according to the LC decoder output in (3.14) and (3.15).

  - The case that LC decoder output for x_i is (3.14);

    From (3.14), if $s_{i,R} > \sqrt{\rho}\{p_1 g_1 + p_2 g_2 + p_3 w\}(\mathcal{R}(x_i) + d)$ or $s_{i,R} < \sqrt{\rho}\{p_1 g_1 + p_2 g_2 + p_3 w\}$ $(\mathcal{R}(x_i) - d)$, the destination can erroneously decode $\mathcal{R}(x_i)$ to $\mathcal{R}(x_i) + 2d$ or $\mathcal{R}(x_i) - 2d$. Also, if $s_{i,I} > \sqrt{\rho}\{p_1 g_1 + p_2 g_2 + p_3 w\}(\mathcal{I}(x_i) + d)$ or $s_{i,I} < \sqrt{\rho}\{p_1 g_1 + p_2 g_2 + p_3 w\}(\mathcal{I}(x_i) - d)$, the destination can erroneously decode $\mathcal{I}(x_i)$ to $\mathcal{I}(x_i) + 2d$ or $\mathcal{I}(x_i) - 2d$. In fact, the SER for all these cases is the same as $P_{e3,2}$.

  - The case that LC decoder output for x_i is (3.15);

    Similar to the previous case, we can obtain the following SER by considering (3.15)

$$
\begin{aligned}
&P_{e3,3} = \\
&E\Bigg[Q\Bigg(d\sqrt{2\rho}\Bigg(p_1 g_1 + \sum_{(i,j)\notin D} p_2 |g_{2,i,j}|^2 + \sum_{(l,k)\notin D} p_3 |f_{2,l,k}|^2 \\
&+ \sum_{(i,j),(l,k)\in D} |\sqrt{p_2} g_{2,i,j} \oplus \sqrt{p_3} f_{l,k}|^2\Bigg) \\
&/(p_1 g_1 + p_2 g_2 + p_3 w)\Bigg)\Bigg]
\end{aligned}
$$

    where $|\sqrt{p_2} g_{2,i,j} \oplus \sqrt{p_3} f_{l,k}|^2$ denotes one of $|\sqrt{p_2} g_{2,i,j} + \sqrt{p_3} f_{l,k}|^2$,

$|\sqrt{p_2}g_{2,i,j} - \sqrt{p_3}f_{l,k}|^2$, $|\sqrt{p_2}g_{2,i,j} + j\sqrt{p_3}f_{l,k}|^2$, and $|\sqrt{p_2}g_{2,i,j} - j\sqrt{p_3}f_{l,k}|^2$ depending on the structure of DOSTBCs like the operator $\circ$ in (3.15). However, all possible $P_{e3,3}$'s depending on $\oplus$ have the same value because each channel coefficient is a complex Gaussian random variable with zero mean. Thus, we can see that $\oplus$ can be replaced with the addition without changing the result.

Finally, the SER for $x_i, i \neq k$ can be expressed as

$$P_{\text{error},i} = \gamma_i P_{e3,2} + \delta_i P_{e3,3}$$

where γ_i and δ_i are constants determined by the constellation and structure of DOSTBCs. Since the LC decoder output for x_i cannot become (3.14) and (3.15) simultaneously, for each $P_{\text{error},i}$, if γ_i is not zero, δ_i should be zero and if δ_i is not zero, γ_i should be zero.

- The SER at the destination when one bit error occurs at the relay (P_{e3}):

  When one-bit error for x_k occurs at the relay, the SER at the destination can be expressed as

$$P_{e3} = \frac{\sum_{i\in\{1,..,L\}} P_{\text{error},i}}{L} = \frac{P_{\text{error},k} + \sum_{i\in\{1,...,L\},i\neq k}^{L} P_{\text{error},i}}{L}$$
$$= \frac{P_{e3,1} + \beta P'_{e3,1} + \gamma_k P_{e3,2} + \sum_{i\in\{1,...,L\},i\neq k}^{L} \gamma_i P_{e3,2} + \sum_{i\in\{1,...,L\},i\neq k}^{L} \delta_i P_{e3,3}}{L}. \tag{3.19}$$

Plugging P_{e1}, P_{e2}, and P_{e3} into (3.9), we can obtain the ROT. In fact, we cannot derive or prove something not because it is complicated but because it is hard or intractable. However, the ROT in high SNR region can be derived by the following approximation.

Since the direct approximation of $P_{e3,1}$ from (3.17) is too complicated, we use the assumption that the effect of noise is negligible in high SNR region and thus (3.12) is approximated as

$$s_{k,R} \approx \sqrt{\rho}\left\{p_1 g_1 + p_2 g_2 + p_3 w\right\}\mathcal{R}(x_k) + 2dp_3 w. \tag{3.20}$$

Then, the error can occur if $s_{k,R} > \sqrt{\rho}\{p_1 g_1 + p_2 g_2 + p_3 w\}(\mathcal{R}(x_k) + d)$ in high SNR region and thus the SER ($P_{e3,1}$) is given as

$$P_{e3,1} \approx \tilde{P}_{e3,1} = \Pr\left(p_1 g_1 + p_2 g_2 < p_3 w\right).$$

By using the PDF and the cumulative distribution function (CDF) of the sum of independent and nonidentical Erlang random variables in [31], $\tilde{P}_{e3,1}$ can be easily calculated.

As SNR increases, $P'_{e3,1}, P_{e3,2}$, and $P_{e3,3}$ in (3.19) become negligible compared to $\tilde{P}_{e3,1}$ and thus P_{e3} becomes close to $\tilde{P}_{e3,1}/L$. Since P_{e2} in (3.11) decreases much faster than P_{e1} in (3.10) as SNR increases, only P_{e1} and $\tilde{P}_{e3,1}/L$ become dominant in the numerator and denominator in (3.9), respectively. Finally, in high SNR region, the ROT in (3.9) is simplified as

$$T = \begin{cases} \frac{M-1}{3}\left[Q^{-1}\left(\frac{P_{e1}}{4\left(1-\frac{1}{\sqrt{M}}\right)\tilde{P}_{e3,1}}\right)\right]^2, & P_{e1} > P_{e2} \\ \infty, & \text{otherwise.} \end{cases} \tag{3.21}$$

3.3.3. Decision of Suboptimal Relay On-Off Threshold in Low SNR Region

In the previous subsection, an ROT was derived only for high SNR region because P_{e3} cannot be derived in low SNR region. Thus, for the whole SNR region including low SNR region, the ROT should be determined.

The ROT in (3.9) is approximated as (3.21) by using the following approximation

$$\frac{P_{e2} - P_{e1}}{4L\left(1 - \frac{1}{\sqrt{M}}\right)(P_{e2} - P_{e3})} \approx \frac{P_{e1}}{4L\left(1 - \frac{1}{\sqrt{M}}\right)P_{e3}} \approx \frac{P_{e1}}{4L\left(1 - \frac{1}{\sqrt{M}}\right)\frac{\tilde{P}_{e3}}{L}}. \tag{3.22}$$

However, the ROT in (3.21) cannot be calculated in low SNR region because $\tilde{P}_{e3,1}/L$ becomes less than P_{e1} as SNR decreases. The argument of $Q^{-1}(\cdot)$ can be greater than 1 when the R-D channel state is not better than the S-D channel state such as (ρ, ρ, ρ). Therefore, a suboptimal ROT for low SNR region is heuristically obtained by giving a bias such as using $\tilde{P}_{e3,1}/L + P_{e1}$ instead of $\tilde{P}_{e3,1}/L$ in (3.22). Thus the ROT in (3.21) can be modified as

$$T = \frac{M-1}{3}\left[Q^{-1}\left(\frac{P_{e1}}{4L\left(1 - \frac{1}{\sqrt{M}}\right)(\frac{\tilde{P}_{e3,1}}{L} + P_{e1})}\right)\right]^2. \tag{3.23}$$

Note that, as SNR increases, the ROT in (3.23) becomes identical to the ROT in (3.21) because P_{e1} goes to zero.

3.3.4. Diversity Analysis

In this subsection, the diversity order of NDF protocol with the proposed relay on-off scheme is derived when DOSTBCs and LC decoding are used.

When the relay is off, the transmit signals in the first and second phases form an OSTBC $\begin{bmatrix} \mathbf{C}_{\mathsf{S},1}(x) & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{\mathsf{S},2}(x) \end{bmatrix}$ and when the relay is on, the transmit signals in the first and second phases form a DOSTBC $\begin{bmatrix} \mathbf{C}_{S,1}(x) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{\mathsf{S},2}(x) & \mathbf{C}_{\mathsf{R},2}(x) \end{bmatrix}$. Since the ranks of their difference matrices are $2M_S$ and $2M_S+M_R$, respectively, and the destination has M_D antennas, in high SNR region, P_{e1} and P_{e2} are proportional to $\rho^{-2M_{\mathsf{S}}M_{\mathsf{D}}}$ and $\rho^{-\{M_{\mathsf{S}}M_{\mathsf{D}}+(M_{\mathsf{S}}+M_{\mathsf{R}})M_{\mathsf{D}}\}}$, respectively. Also, $P_{e1}/\left\{4\left(1-1/\sqrt{M}\right)\tilde{P}_{e3,1}\right\}$ in (3.21) can be approximated as $c\rho^{-2M_{\mathsf{S}}M_{\mathsf{D}}}$, where c is a positive constant.

Since $Q^{-1}(x) \approx \sqrt{w-\ln(2\pi w)}$, where $w = -2\ln x$ [32] in high SNR region, the ROT in (3.21) can be approximated as

$$\begin{aligned} T &\approx \frac{M-1}{3}\left\{Q^{-1}\left(\frac{c}{\rho^{2M_{\mathsf{S}}M_{\mathsf{D}}}}\right)\right\}^2 \\ &= \frac{M-1}{3}\left[-2\ln\frac{c}{\rho^{2M_{\mathsf{S}}M_{\mathsf{D}}}} - \ln\left\{2\pi\left(-2\ln\frac{c}{\rho^{2M_{\mathsf{S}}M_{\mathsf{D}}}}\right)\right\}\right] \approx \frac{M-1}{3}\ln\rho^{4M_{\mathsf{S}}M_{\mathsf{D}}}. \end{aligned} \tag{3.24}$$

For simplicity, let $z = p_1\sum_{i=1}^{M_{\mathsf{S}}}\sum_{j=1}^{M_{\mathsf{R}}}|h_{\mathsf{SR},i,j}|^2$ and $g(z)$ be the PDF of z. Then the SER in (3.8) at the destination is rewritten as

$$P_e(T) = P_{e1}\int_0^{\frac{T}{\rho}} g(z)dz + P_{e2}\int_{\frac{T}{\rho}}^{\infty}\left\{1-4L\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3\rho z}{M-1}}\right)\right\}g(z)dz$$

$$+ P_{e3} \int_{\frac{T}{\rho}}^{\infty} 4L\left(1-\frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3\rho z}{M-1}}\right) g(z)dz. \tag{3.25}$$

Now, we derive the diversity order for each term in (3.25). Since $e^{-\lambda\frac{T}{\rho}} = \sum_{m=0}^{\infty}\left(-\lambda\frac{T}{\rho}\right)^m/m!$, the CDF of z is given as

$$G(z) = 1 - \sum_{n=0}^{M_{\mathsf{S}}M_{\mathsf{R}}-1} \frac{e^{-\lambda z}(\lambda z)^n}{n!} = e^{-\lambda z}\sum_{n=M_{\mathsf{S}}M_{\mathsf{R}}}^{\infty}\frac{(\lambda z)^n}{n!} \tag{3.26}$$

where $\lambda = 1/p_1\sigma_{\mathsf{SR}}^2$.

The first term in (3.25) can be given as

$$P_{e1}\int_0^{\frac{T}{\rho}} g(z)dz = P_{e1}G\left(\frac{T}{\rho}\right) = P_{e1}\left\{\sum_{n=M_{\mathsf{S}}M_{\mathsf{R}}}^{\infty}\sum_{m=0}^{\infty}\left(\lambda\frac{T}{\rho}\right)^n\left(-\lambda\frac{T}{\rho}\right)^m/m!n!\right\} \tag{3.27}$$

where $(\lambda T/\rho)^{M_{\mathsf{S}}M_{\mathsf{R}}} P_{e1}/(M_{\mathsf{S}}M_{\mathsf{R}})!$ is dominant because $T/\rho << 1$ as ρ increases. Using (3.24) and (3.27), the diversity order of the first term in (3.25) is obtained as $2M_{\mathsf{S}}M_{\mathsf{D}} + M_{\mathsf{S}}M_{\mathsf{R}}$.

As SNR increases, $4L\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3\rho z}{M-1}}\right)$ goes to zero and thus $P_{e2}\int_{\frac{T}{\rho}}^{\infty} g(z)dz$ becomes dominant in the second term in (3.25). Therefore, it is clear that the diversity order of the second term in (3.25) is $2M_{\mathsf{S}}M_{\mathsf{D}} + M_{\mathsf{R}}M_{\mathsf{D}}$ because $P_{e2}\int_{\frac{T}{\rho}}^{\infty} g(z)dz = P_{e2}(1-G(T/\rho))$.

Let $K = 4L(1-1/\sqrt{M})$ and $\delta = 3/(M-1)$. Then, the third term in (3.25) is upper bounded by

$$P_{e3}\int_{\frac{T}{\rho}}^{\infty} KQ\left(\sqrt{\delta\rho z}\right) g(z)du$$

$$\leq P_{e3}\int_{\frac{T}{\rho}}^{\infty} K e^{\frac{-\delta u\rho}{2}} g(z)dz \text{ (by Chernoff bound)}$$

$$= \frac{P_{e3}K\lambda^{M_{\mathsf{S}}M_{\mathsf{R}}-1}e^{-\left(\frac{\delta\rho}{2}+\lambda\right)\frac{T}{\rho}}}{(M_{\mathsf{S}}M_{\mathsf{R}}-1)!}\left\{\frac{1}{\frac{\delta\rho}{2}+\lambda}\left(\frac{T}{\rho}\right)^{M_{\mathsf{S}}M_{\mathsf{R}}-1}+\frac{(M_{\mathsf{S}}M_{\mathsf{R}}-1)}{\left(\frac{\delta\rho}{2}+\lambda\right)^{2}}\left(\frac{T}{\rho}\right)^{M_{\mathsf{S}}M_{\mathsf{R}}-2}\right.$$

$$\left.+\frac{(M_{\mathsf{S}}M_{\mathsf{R}}-1)(M_{\mathsf{S}}M_{\mathsf{R}}-2)}{\left(\frac{\delta\rho}{2}+\lambda\right)^{3}}\left(\frac{T}{\rho}\right)^{M_{\mathsf{S}}M_{\mathsf{R}}-3}+\cdots+\frac{(M_{\mathsf{S}}M_{\mathsf{R}}-1)!}{\left(\frac{\delta\rho}{2}+\lambda\right)^{M_{\mathsf{S}}M_{\mathsf{R}}}}\right\}. \tag{3.28}$$

Note that the factor $e^{-\left(\frac{\delta\rho}{2}+\lambda\right)\frac{T}{\rho}}\left(\frac{T}{\rho}\right)^{M_{\mathsf{S}}M_{\mathsf{R}}-1}/(\frac{\delta\rho}{2}+\lambda)$ of the first term in (3.28) becomes dominant as ρ increases. Plugging (3.24) into (3.28), this factor becomes as

$$\frac{e^{-\left(\frac{\delta\rho}{2}+\lambda\right)\frac{T}{\rho}}\left(\frac{T}{\rho}\right)^{M_{\mathsf{S}}M_{\mathsf{R}}-1}}{\frac{\delta\rho}{2}+\lambda}=\frac{e^{-\left(\frac{\ln\rho^{4M_{\mathsf{S}}M_{\mathsf{D}}}}{2}+\lambda\frac{\ln\rho^{4M_{\mathsf{S}}M_{\mathsf{D}}}}{\rho}\right)}}{\frac{\delta\rho}{2}+\lambda}\left(\frac{\ln\rho^{4M_{\mathsf{S}}M_{\mathsf{D}}}}{\rho}\right)^{(M_{\mathsf{S}}M_{\mathsf{R}}-1)}.$$

Since $\lambda\frac{\ln\rho^{4M_{\mathsf{S}}M_{\mathsf{D}}}}{\rho}$ in $e^{-\left(\frac{\ln\rho^{4M_{\mathsf{S}}M_{\mathsf{D}}}}{2}+\lambda\frac{\ln\rho^{4M_{\mathsf{S}}M_{\mathsf{D}}}}{\rho}\right)}$ becomes negligible as SNR increases, the diversity order of (3.28) is given as $2M_{\mathsf{S}}M_{\mathsf{D}}+M_{\mathsf{S}}M_{\mathsf{R}}$.

In conclusion, the diversity order d_{LC} of NDF protocol with the LC decoding and the proposed relay on-off scheme is given as

$$d_{LC}=\min\left(2M_{\mathsf{S}}M_{\mathsf{D}}+M_{\mathsf{S}}M_{\mathsf{R}},2M_{\mathsf{S}}M_{\mathsf{D}}+M_{\mathsf{R}}M_{\mathsf{D}}\right).$$

Note that the diversity order $2M_{\mathsf{S}}M_{\mathsf{D}}+\min(M_{\mathsf{S}}M_{\mathsf{R}},M_{\mathsf{R}}M_{\mathsf{D}})$ corresponds to the full diversity order of NDF protocol with SAS [10].

3.4. Numerical Analysis

For the simulation, it is assumed that the transmit signal power in the first phase is the same as the sum of transmit signal powers from the source and relay in the second phase, and the transmit signal power of the relay is the same as that of the source in the second phase. (x, y, z) denotes the average received SNRs in dB of S-D, S-R, and R-D channels, respectively. For example, $(\rho, \rho, \rho + 6)$ means that the average received SNR of R-D channel is larger than those of S-R and S-D channels by 6 dB. For all simulations, 16QAM is used.

To confirm the diversity order, when $M_S, M_R, M_D \geq 2$, simulation must be performed in very high SNR region, which requires too long time. Instead, simulation has been performed for the single-antenna case ($M_S = M_R = M_D = 1$). For $M_S = M_R = M_D = 1$, the transmitted OSTBCs are given as

$$\mathbf{C}_{\mathsf{S},1}(\mathbf{x}) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \mathbf{C}_{\mathsf{S},2}(\mathbf{x}) = \begin{bmatrix} x_1 \\ -x_2^* \end{bmatrix}, \mathbf{C}_{\mathsf{R},2}(\mathbf{x}) = \begin{bmatrix} x_2 \\ x_1^* \end{bmatrix}$$

where the combination of $\mathbf{C}_{\mathsf{S},2}$ and $\mathbf{C}_{\mathsf{R},2}$ forms an Alamouti code in the second phase. The ROT for the single-antenna case can also be easily obtained from (3.21) and (3.23).

Figs. 3.2 and 3.3 compare the performance of NDF protocol with various relay schemes. We do not consider the case that the S-R channel state is better than the other channel states because the relay always tends to be on. The performance of the proposed optimal and suboptimal relay on-

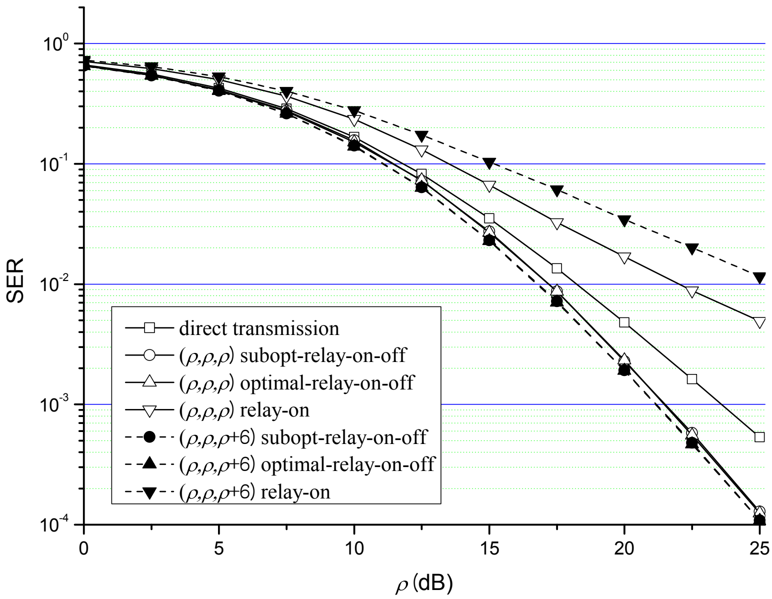

Figure 3.2: Performance comparison of NDF protocol with various relay schemes for single-antenna case using 16QAM under various R-D channel states.

off schemes and the conventional relay scheme are denoted by 'optimal-relay-on-off', 'subopt-relay-on-off', and 'relay-on', respectively. In other words, 'relay-on' means that the relay always transmits signal. The 'direct transmission' implies that the relay is always off. The optimal relay on-off scheme uses the optimal ROT which is determined by extensive simulation and the suboptimal relay on-off scheme uses the ROT in (3.23).

Fig. 3.2 shows that the SER performance of the relay-on scheme becomes worse as the R-D channel state becomes better and thus, in this case, the relay on-off scheme is vital to improve the LC decoding performance. For the case of $(\rho + 6, \rho, \rho)$ in Fig. 3.3, it can be seen that the

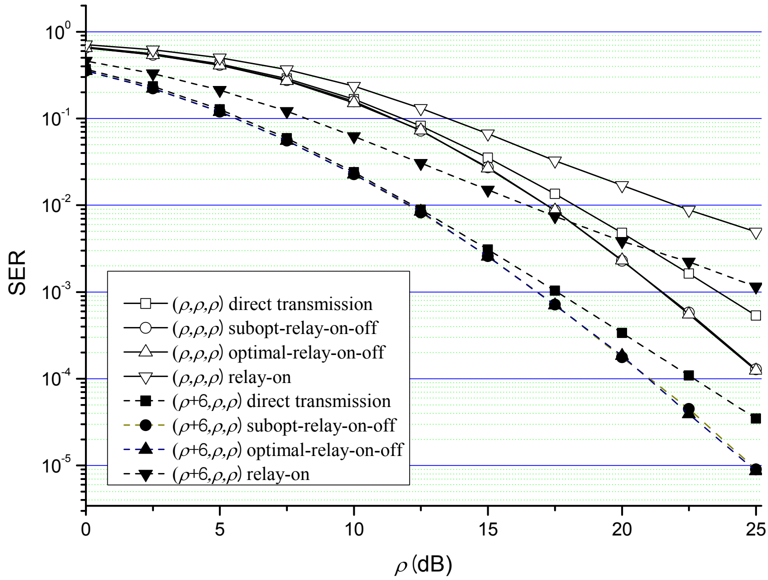

Figure 3.3: Performance comparison of NDF protocol with various relay schemes for single-antenna case using 16QAM under various S-D channel states.

performance of the relay on-off scheme is almost identical to that of the direct transmission until $\rho = 12.5$ dB, which is because $P_{e1} \leq P_{e2}$ and the relay is always off. Therefore, the ROT should be infinite, as given by (3.9). Also, we can see that the analytical diversity results in Section 3.4 are well matched with the simulation results in Figs. 3.2 and 3.3, and the suboptimal ROT works well in low SNR region as well as in high SNR region.

Figs. 3.4 and 3.5 compare the performance of NDF protocol with various relay schemes when $M_S = M_R = M_D = 2$ and the DOSTBC in (3.16) is used. In Fig. 3.4, since the diversity effect appears gradually from $\rho = 12.5$

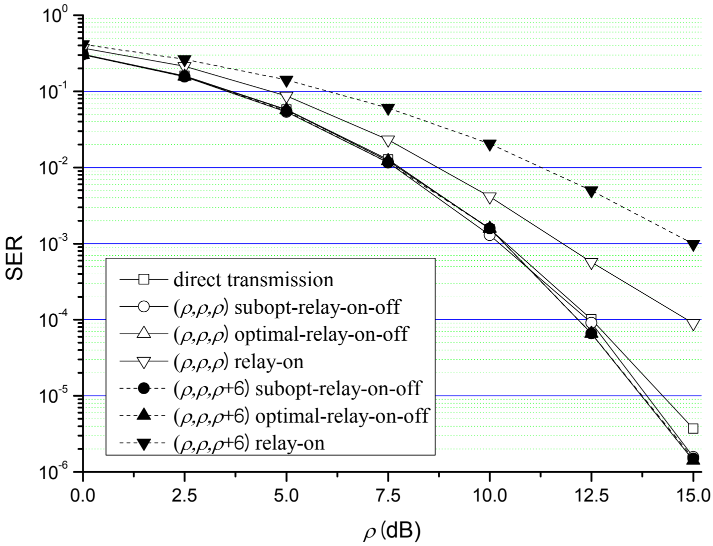

Figure 3.4: Performance comparison of NDF protocol with various relay schemes for two-antenna case using 16QAM under various R-D channel states.

dB, it can be expected that the diversity order of relay on-off scheme will reach 12 in higher SNR region. For the case of $(\rho + 6, \rho, \rho)$ in Fig. 3.5, it can be seen that the performance of the relay on-off scheme is almost identical to that of the direct transmission, which implies that the ROT is infinite. Since the diversity effect will appear as SNR increases, it is expected that the relay becomes helpful as a virtual antenna to increase the diversity order in higher SNR region in this case. Also, the simulation results show that the suboptimal ROT works well in low SNR region as well as in high SNR region.

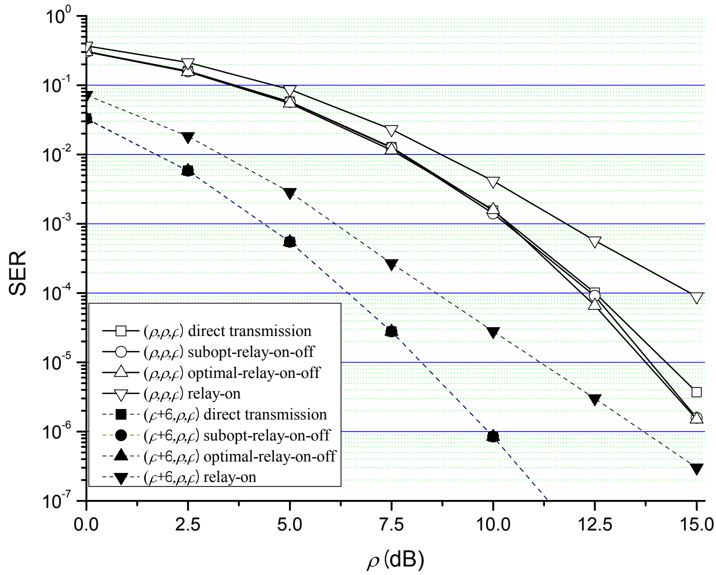

Figure 3.5: Performance comparison of NDF protocol with various relay schemes for two-antenna case using 16QAM under various S-D channel states.

3.5. Conclusion

In this chapter, an ROT is analytically derived for NDF protocol with DOSTBCs and SAS in high SNR region, where LC decoding is considered because it has low complexity and good performance. Through the diversity analysis, it is confirmed that the NDF protocol with the proposed relay on-off scheme can achieve the full diversity. In low SNR region, the suboptimal ROT is provided and the simulation results confirm that this suboptimal ROT works well in whole SNR region. It is left as a future work to derive the optimal ROT for the cooperative communication networks with multiple relays by using a similar method to one proposed in this chapter.

Chapter 4. New Interference Alignment Scheme Aided by Relays for Quasi-Static X Channels

4.1. Introduction

As the number of multimedia mobile devices such as smart phone is dramatically increasing, data communication with very high spectral efficiency must be guaranteed to support various mobile multimedia services. For that purpose, multi-user interference must be mitigated and hence many interference management schemes have been proposed [33]–[36].

Recently, IA has attracted much attention for interference management. Especially, IA schemes for general K-user interference channel and $M \times N$ X channel were proposed in [5, 6], respectively. By extending the symbols of transmitters in time domain, they can asymptotically achieve the maximum DoF for each channel. Also, to achieve the maximum DoF, a relay-aided approach was investigated for the 3-user interference channel [37], which can achieve DoF 3/2. However, these schemes assume that the channel state varies at every transmission time. Since all transmitters and receivers need global CSI, which requires a lot of feedback information, these schemes may not be practical for the time-varying channel. Therefore, research on IA in the slow fading environment has been done from the practical viewpoint [37, 38].

In order to implement IA scheme in the slow fading environment, multiple channels such as multiple carriers or multiple antennas can be used. However, such resources are sometimes limited or hard to implement. Therefore, IA scheme with time extension may be preferred to support multiple users.

Nourani et al. proposed relay-aided IA schemes for the quasi-static X channel and interference channel in [7, 8], respectively. They used full-duplex relays with some memories and proved that the beamforming vectors for IA can be designed for the quasi-static fading channel environment. These schemes imply that IA can be practically implemented by using time extension. However, since the transmitted signal from a relay can be fed back to the relay as a self interference in full-duplex mode, called echo, the half-duplex relays are widely used instead of the full-duplex relays.

In this chapter, a new simple IA scheme with a full-duplex relay and an IA scheme with two half-duplex relays are proposed for the quasi-static X channel. Comparing with the scheme in [7], the first proposed scheme reduces the hardware complexity by using only one memory and the second one uses two half-duplex relays instead of the full-duplex relay. Therefore, the proposed schemes can be thought more practical than the scheme in [7]. By verifying that the desired and interference signal vectors are linearly independent with each other, the maximum DoF for the proposed IA schemes is derived. Also, it is shown that the proposed IA schemes are applicable to the $2 \times M$ X channel by using the reciprocity.

4.2. Preliminaries: X Channel and Interference Alignment

In this section, X channel and IA scheme in [6] are described. Assuming that each node has single antenna, the $M \times N$ X channel can be defined as a communication channel with M transmitters and N receivers for sending MN independent messages, where each message is transmitted from each transmitter to each receiver. Fig. 4.1 shows the 3×2 X channel, where W_{ji} denotes the message from the transmitter i to the receiver j. In general, the $M \times N$ X channel can be thought that each transmitter has N messages to be transmitted to all receivers and each receiver receives M messages from all transmitters.

In [6], an IA scheme for X channel was proposed and it was proved that the maximum DoF of the $M \times N$ X channel is $MN/(M+N-1)$. However, in general, an infinite symbol extension is required to achieve this maximum DoF but for the $M \times 2$ X channel, DoF $2M/(M+1)$ can be obtained by using $M+1$ symbol extension.

In this chapter, we focus on the $M \times 2$ X channel with relays. It is well known that the symbol extension and the design of beamforming vectors are essential for the IA schemes, where the beamforming vectors make the interference signals properly aligned for the receivers. The beamforming vectors for IA can be designed by using the channel matrices as follows. First, we should design linearly independent beamforming vectors $\mathbf{b}_{11}$

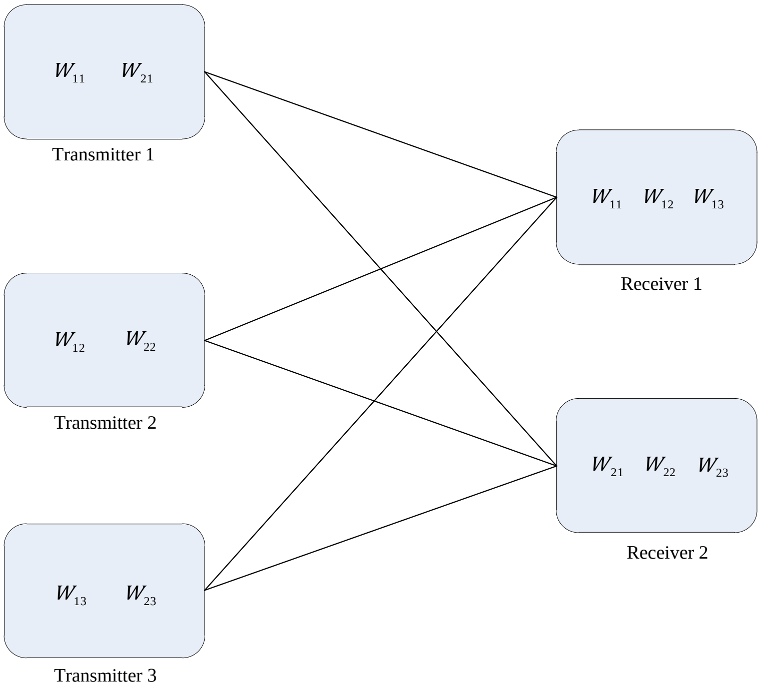

Figure 4.1: 3×2 X channel.

and $\mathbf{b}_{21}$ for the signals from the transmitter 1 to the receivers 1 and 2, respectively. Then, the other beamforming vectors $\mathbf{b}_{1i}$ and $\mathbf{b}_{2i}$ from the transmitter i to the receivers 1 and 2 are designed by using the following relations [6]

$$\mathbf{H}_{2i}\mathbf{b}_{1i} = \mathbf{H}_{21}\mathbf{b}_{11}, \ i = 2, \ldots, M \tag{4.1}$$

$$\mathbf{H}_{1i}\mathbf{b}_{2i} = \mathbf{H}_{11}\mathbf{b}_{21}, \ i = 2, \ldots, M \tag{4.2}$$

where $\mathbf{b}_{ji}$ and $\mathbf{H}_{ji}$ denote the $(M+1) \times 1$ beamforming vector and the $(M+1) \times (M+1)$ channel matrix between the transmitter i and the receiver j, respectively. The equality in (4.1) and (4.2) means that two vectors in the left and right hand sides of the equality are placed in the

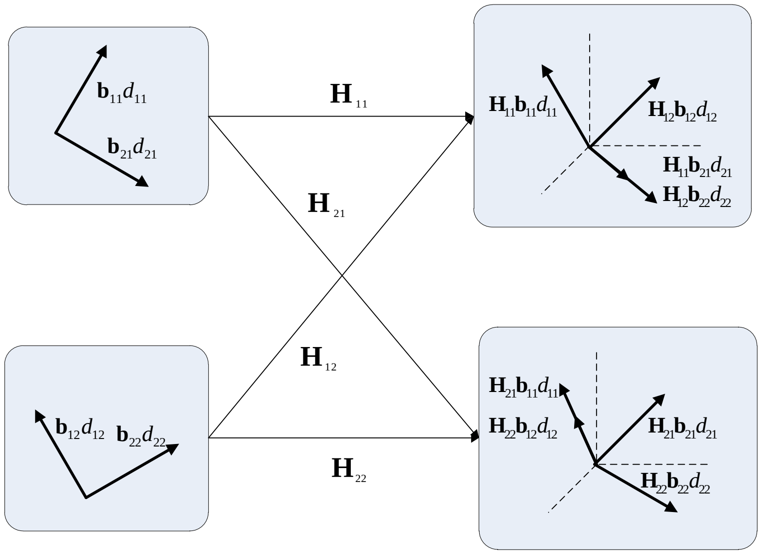

Figure 4.2: Interference alignment for the 2×2 X channel.

same direction in the vector space.

The received signal spaces at each receiver can be given as

$$\Gamma_1 = \begin{bmatrix} \mathbf{H}_{11}\mathbf{b}_{11} & \mathbf{H}_{12}\mathbf{b}_{12} & \cdots & \mathbf{H}_{1M}\mathbf{b}_{1M} & \mathbf{H}_{11}\mathbf{b}_{21} \end{bmatrix} \tag{4.3}$$

$$\Gamma_2 = \begin{bmatrix} \mathbf{H}_{21}\mathbf{b}_{21} & \mathbf{H}_{22}\mathbf{b}_{22} & \cdots & \mathbf{H}_{2M}\mathbf{b}_{2M} & \mathbf{H}_{21}\mathbf{b}_{11} \end{bmatrix}. \tag{4.4}$$

If these matrices have the full rank, each receiver can detect all desired signal vectors from M transmitters by zero-forcing filter, which is called perfect IA in this chapter. In [6], it was proved that these matrices have the full rank, which guarantees $2M/(M+1)$ DoF. Fig. 4.2 describes IA scheme for the 2×2 X channel, where d_{ji} is the data stream from the transmitter i to the receiver j. By using the beamforming vectors $\mathbf{b}_{ji}$, each transmitter can align its signals along the directions different from those of interference signals so that the desired signals are linearly independent

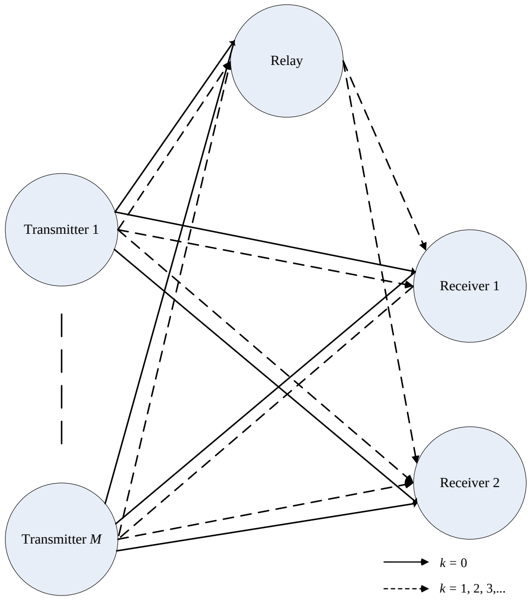

Figure 4.3: System models for the proposed schemes over the $M \times 2$ X channel: Aided by a full-duplex relay.

from the interference signals at each receiver. However, in [6], it is assumed that every channel is time-varying and the equivalent channel matrix has the diagonal form. Since the quasi-static X channel with relays is assumed in this chapter, the analysis in [6] cannot be applied. Therefore, it should be proved that Γ_1 and Γ_2 have the full rank for the proposed schemes.

4.3. The Proposed Schemes and System Models

In this section, two IA schemes with relays are proposed and their corresponding system models are given.

4.3.1. Two Proposed Schemes

- A simple IA scheme with a full-duplex relay:

Actually, this scheme described in Fig. 4.3 is the low-complexity version of IA scheme with a full-duplex relay in [7], that is, the relay in the proposed scheme receives signals in every time slot and amplifies and forwards them to the receivers in the next time slot. In the IA scheme in [7], the relay stores all the received signals which are needed for symbol extension, and amplifies and adds all the received signals with different amplification gains and forwards them to the receivers. Thus, the IA scheme in [7] can have some performance gain by optimizing the amplification gain for each received signal but this optimization is complicated and requires lots of memory to store all the received signals. In the proposed IA scheme, the maximum DoF can be obtained without storing all the received signals, which will be proved in Section 4.4. Therefore, the proposed scheme can be a good simple alternative method for the IA scheme in [7].

- An IA scheme with two half-duplex relays:

  It is hard to implement a full-duplex relay because the power of transmit signal is much stronger than that of the received signal at the relay and thus the transmit signal can cause a strong self-interference at the relay. Therefore, an IA scheme with half-duplex relays is preferred from the practical viewpoint and thus, we propose an IA scheme with two half-duplex relays.

  By using two half-duplex relays as in Fig. 4.4, we can obtain the same effect as that of the IA scheme in [7] which has to store all the

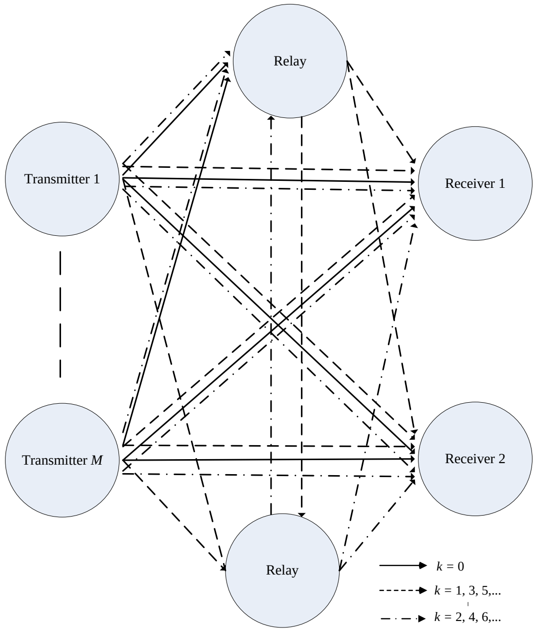

Figure 4.4: System models for the proposed schemes over the $M \times 2$ X channel: Aided by two half-duplex relays.

received signals and forward them. If two half-duplex relays transmit and receive by turns in amplify-and-forward (AF) mode, the received signal at each relay in each time slot contains all the previous and current received signals. If this scheme achieves the maximum DoF for the $M \times 2$ X channel, it can be considered more practical than the other schemes with a full-duplex relay. In the next section, it will be proved that the proposed IA scheme with two half-duplex relays can obtain the maximum DoF for the $M \times 2$ X channel.

4.3.2. System Models for the Proposed Schemes

It is assumed that the number of transmitters is M and the number of receivers is two. It is also assumed that each node has one antenna and each channel is quasi-static Rayleigh fading channel. All transmitters and receivers know all the channel state information. Figs. 4.3 and 4.4 show the system models for the two proposed relay-aided IA schemes.

- The system model using a full-duplex relay in Fig. 4.3:

  In this model, a full-duplex relay operates in AF mode. Let $y_r(k)$ and $y_j(k)$ be the received signals at the relay and the receiver j in the time slot k, $k = 0, 1, \cdots, M$. Then the system model is given as

$$y_r(k) = \sum_{i=1}^{M} h_{ri} x_i(k) + n_r(k)$$
$$y_j(k) = \sum_{i=1}^{M} h_{ji} x_i(k) + u(k) h_{jr} y_r(k-1) + n_j(k), \ j = 1, 2$$

  where $u(k)$ denotes the relay gain in AF mode with $u(0) = 0$, and h_{ri} and h_{jr} denote the channel coefficients between the transmitter i and the relay and between the relay and the receiver j, respectively. $n_r(k)$ and $n_j(k)$ are the additive complex white Gaussian noises at the relay and receiver j, respectively. $x_i(k)$ denotes the transmitted signal from the transmitter i, which is expressed as $x_i(k) = b_{1i,k} d_{1i} + b_{2i,k} d_{2i}$, where $b_{ji,k}$ is the kth element of the beamforming vector $\mathbf{b}_{ji}$ and d_{ji} is the desired data symbol from the transmitter i to the receiver j. For simplicity, it is assumed that h_{ri} and h_{jr} are independent continuous random variables and $n_r(k)$ and $n_j(k)$ are independent complex white Gaussian random variables with the distribution $CN(0, 1)$.

  This model is simpler than that in [7] because the receiver j receives only $x_i(k)$ and $y_r(k-1)$ at time k. In this model, each transmitter uses $M+1$ symbol extension to transmit two data streams because we have M transmitters, which implies that the maximum DoF $2M/(M+1)$ can be achieved. The achievability of the DoF will be proved in Section 4.4.

- The system model using two half-duplex relays in Fig. 4.4:

  In this model, two half-duplex relays are used between M transmitters and two receivers. Since two half-duplex relays transmit and receive signals by turns, the received signals at the relays are given as

$$y_{r_1}(k) = \sum_{i=1}^{M} h_{r_1 i} x_i(k) + g_2 u(k) y_{r_2}(k-1) + n_{r_1}(k),$$

$$k = 0, 2, 4, \ldots, \text{ at relay 1}$$

$$y_{r_2}(k) = \sum_{i=1}^{M} h_{r_2 i} x_i(k) + g_1 u(k) y_{r_1}(k-1) + n_{r_2}(k),$$

$$k = 1, 3, 5, \ldots, \text{ at relay 2}$$

and the received signal at each receiver is also given as

$$y_j(k) = \begin{cases} \sum_{i=1}^{M} h_{ji} x_i(k) + u(k) h_{jr_2} y_{r_2}(k-1) + n_j(k), \\ \qquad\qquad\qquad \text{for } k = 0, 2, 4, \ldots \\ \sum_{i=1}^{M} h_{ji} x_i(k) + u(k) h_{jr_1} y_{r_1}(k-1) + n_j(k), \\ \qquad\qquad\qquad \text{for } k = 1, 3, 5, \ldots \end{cases}$$

where $j = 1, 2$, $u(0) = 0$, and $h_{r_1 i}$, h_{jr_1}, $h_{r_2 i}$, and h_{jr_2} denote the channel coefficients from the transmitter i to the relay 1, from the relay 1 to the receiver j, from the transmitter i to the relay 2, and from the relay 2 to the receiver j, respectively and g_1 and g_2 are the channel coefficients from the relay 1 to 2 and from the relay 2 and 1, respectively. All the channel coefficients are statistically independent with some continuous distributions. n_{r1} and n_{r2} are the additive complex white Gaussian noises at the relays 1 and 2, respectively, with the distribution $CN(0, 1)$.

In this model, the transmitters transmit new signals at each time slot and each relay operates in AF mode (non-orthogonal AF). However, if one relay transmits the signal, the other should receive. In other words, they transmit and receive the signals by turns. Since a relay receives signals from the transmitters and the other relay, this signal contains all the previous and current received signals. Note that each transmitter also uses $M + 1$ symbol extension to transmit two data streams.

4.4. Achievability of the Proposed IA Schemes

In this section, we investigate the equivalent channel matrix of each system model and show that the perfect IA can be achieved by the proposed relay-aided IA schemes.

4.4.1. IA Scheme with a Full-Duplex Relay

The equivalent channel matrix between the transmitter i and the receiver j in Fig. 4.3 is given as

$$\mathbf{H}_{ji} = h_{ji} \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ \frac{u(1)h_{ri}h_{jr}}{h_{ji}} & 1 & 0 & 0 & \cdots & 0 \\ 0 & \frac{u(2)h_{ri}h_{jr}}{h_{ji}} & 1 & 0 & \cdots & 0 \\ \vdots & 0 & \frac{u(3)h_{ri}h_{jr}}{h_{ji}} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \frac{u(M)h_{ri}h_{jr}}{h_{ji}} & 1 \end{bmatrix}. \quad (4.5)$$

In this $(M+1)\times(M+1)$ matrix, the main diagonal elements and subdiagonal elements are not zero but the other elements are all zero. Its inverse matrix $\mathbf{H}_{ji}^{-1}$ is given as

$$\mathbf{H}_{ji}^{-1} = h_{ji}^{-1} \times$$
$$\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ -\frac{u(1)h_{ri}h_{jr}}{h_{ji}} & 1 & 0 & 0 & \cdots & 0 \\ \frac{u(1)u(2)(h_{ri}h_{jr})^2}{h_{ji}^2} & -\frac{u(2)h_{ri}h_{jr}}{h_{ji}} & 1 & 0 & \cdots & 0 \\ -\frac{u(1)u(2)u(3)(h_{ri}h_{jr})^3}{h_{ji}^3} & \frac{u(2)u(3)(h_{ri}h_{jr})^2}{h_{ji}^2} & -\frac{u(3)(h_{ri}h_{jr})}{h_{ji}} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ (-1)^M \frac{\prod_{k=1}^{M} u(k)(h_{ri}h_{jr})^M}{h_{ji}^M} & (-1)^{M-1} \frac{\prod_{k=2}^{M} u(k)(h_{ri}h_{jr})^{M-1}}{h_{ji}^{M-1}} & \cdots & \cdots & -\frac{u(M)h_{ri}h_{jr}}{h_{ji}} & 1 \end{bmatrix}.$$

The received signal space Γ_1 for the receiver 1 can be given as

$$\begin{aligned} \Gamma_1 &= \begin{bmatrix} \mathbf{H}_{11}\mathbf{b}_{11} & \mathbf{H}_{12}\mathbf{H}_{22}^{-1}\mathbf{H}_{21}\mathbf{b}_{11} & \cdots & \mathbf{H}_{1M}\mathbf{H}_{2M}^{-1}\mathbf{H}_{21}\mathbf{b}_{11} & \mathbf{H}_{11}\mathbf{b}_{21} \end{bmatrix} \\ &= \begin{bmatrix} \tilde{\mathbf{b}}_{11} & \mathbf{H}_{12}\mathbf{H}_{22}^{-1}\mathbf{b}'_{11} & \cdots & \mathbf{H}_{1M}\mathbf{H}_{2M}^{-1}\mathbf{b}'_{11} & \tilde{\mathbf{b}}_{21} \end{bmatrix} \end{aligned} \quad (4.6)$$

where $\tilde{\mathbf{b}}_{11} = \mathbf{H}_{11}\mathbf{b}_{11}$, $\mathbf{b}'_{11} = \mathbf{H}_{21}\mathbf{b}_{11}$, and $\tilde{\mathbf{b}}_{21} = \mathbf{H}_{11}\mathbf{b}_{21}$.

For simplicity, let $u(k) = 1$ and $\tilde{h}_{ji} = h_{ri}h_{jr}/h_{ji}$. Even though we set $u(k) = 1$, by normalizing the relay power with total power, the power constraint can be easily satisfied. Therefore, this assumption does not effect on the proof of the perfect IA. Then, $\mathbf{H}_{1i}\mathbf{H}_{2i}^{-1}$ is given as

$$\mathbf{H}_{1i}\mathbf{H}_{2i}^{-1} = \frac{h_{1i}}{h_{2i}}\times$$
$$\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ \tilde{h}_{1i}-\tilde{h}_{2i} & 1 & 0 & 0 & \cdots & 0 \\ -\tilde{h}_{1i}\tilde{h}_{2i}+\tilde{h}_{2i}^2 & \tilde{h}_{1i}-\tilde{h}_{2i} & 1 & 0 & \cdots & 0 \\ \tilde{h}_{1i}\tilde{h}_{2i}^2-\tilde{h}_{2i}^3 & -\tilde{h}_{1i}\tilde{h}_{2i}+\tilde{h}_{2i}^2 & \tilde{h}_{1i}-\tilde{h}_{2i} & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & 0 \\ (-1)^{M-1}(\tilde{h}_{1i}\tilde{h}_{2i}^{M-1}-\tilde{h}_{2i}^M) & (-1)^{M-2}(\tilde{h}_{1i}\tilde{h}_{2i}^{M-2}-\tilde{h}_{2i}^{M-1}) & \cdots & \cdots & \tilde{h}_{1i}-\tilde{h}_{2i} & 1 \end{bmatrix}.$$

The ith column Γ_{1i} of Γ_1 in (4.6), $i \neq 1, M+1$, is given as

$$\begin{aligned}\Gamma_{1i} &= \mathbf{H}_{1i}\mathbf{H}_{2i}^{-1}\mathbf{b}'_{11} \\ &= \frac{h_{1i}}{h_{2i}}\begin{bmatrix} b'_{11,1} \\ (\tilde{h}_{1i}-\tilde{h}_{2i})b'_{11,1}+b'_{11,2} \\ (-\tilde{h}_{1i}\tilde{h}_{2i}+\tilde{h}_{2i}^2)b'_{11,1}+(\tilde{h}_{1i}-\tilde{h}_{2i})b'_{11,2}+b'_{11,3} \\ (\tilde{h}_{1i}\tilde{h}_{2i}^2-\tilde{h}_{2i}^3)b'_{11,1}+(-\tilde{h}_{1i}\tilde{h}_{2i}+\tilde{h}_{2i}^2)b'_{11,2}+(\tilde{h}_{1i}-\tilde{h}_{2i})b'_{11,3}+b'_{11,4} \\ \vdots \\ \sum_{k=1}^{M}(-1)^{k-1}(\tilde{h}_{1i}\tilde{h}_{2i}^{k-1}-\tilde{h}_{2i}^k)b'_{11,M+1-k}+b'_{11,M+1} \end{bmatrix}\end{aligned}$$

where $b'_{11,k}$ is the kth element of $\mathbf{b}'_{11}$.

In order to confirm that Γ_1 has full rank, it is sufficient to show that $|\Gamma_1|$ is not zero with probability 1. Now, by keeping the highest order term in $\tilde{h}_{2i}$ and removing all other terms in each element of Γ_1, we obtain $\tilde{\Gamma}_1$ as

$\tilde{\Gamma}_1 =$

$$\begin{bmatrix} \tilde{b}_{11,1} & \frac{h_{12}}{h_{22}} b'_{11,1} & \frac{h_{13}}{h_{23}} b'_{11,1} & \cdots & \frac{h_{1M}}{h_{2M}} b'_{11,1} & \tilde{b}_{21,1} \\ \tilde{b}_{11,2} & -\frac{h_{12}}{h_{22}} \tilde{h}_{22} b'_{11,1} & -\frac{h_{13}}{h_{23}} \tilde{h}_{23} b'_{11,1} & \cdots & -\frac{h_{1M}}{h_{2M}} \tilde{h}_{2M} b'_{11,1} & \tilde{b}_{21,2} \\ \tilde{b}_{11,3} & \frac{h_{12}}{h_{22}} \tilde{h}_{22}^2 b'_{11,1} & \frac{h_{13}}{h_{23}} \tilde{h}_{23}^2 b'_{11,1} & \cdots & \frac{h_{1M}}{h_{2M}} \tilde{h}_{2M}^2 b'_{11,1} & \tilde{b}_{21,3} \\ \tilde{b}_{11,4} & -\frac{h_{12}}{h_{22}} \tilde{h}_{22}^3 b'_{11,1} & -\frac{h_{13}}{h_{23}} \tilde{h}_{23}^3 b'_{11,1} & \cdots & -\frac{h_{1M}}{h_{2M}} \tilde{h}_{2M}^3 b'_{11,1} & \tilde{b}_{21,4} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \tilde{b}_{11,M+1} & (-1)^{M-1} \frac{h_{12}}{h_{22}} \tilde{h}_{22}^M b'_{11,1} & (-1)^{M-1} \frac{h_{13}}{h_{23}} \tilde{h}_{23}^M b'_{11,1} & \cdots & (-1)^{M-1} \frac{h_{1M}}{h_{2M}} \tilde{h}_{2M}^M b'_{11,1} & \tilde{b}_{21,M+1} \end{bmatrix} \tag{4.7}$$

where $\tilde{b}_{11,k}$ and $\tilde{b}_{12,l}$ denote the kth element of $\tilde{\mathbf{b}}_{11}$ and the lth element of $\tilde{\mathbf{b}}_{12}$, respectively. Then the determinant of $\tilde{\Gamma}_1$ is given as

$$|\tilde{\Gamma}_1| = \tilde{b}_{11,1} C_{11} - \frac{h_{12}}{h_{22}} b'_{11,1} C_{12} + \cdots + (-1)^{M+1} \tilde{b}_{12,1} C_{1(M+1)} \tag{4.8}$$

where C_{ij} is the cofactor, which is the determinant of the square matrix obtained after deleting the ith row and the jth column of $\tilde{\Gamma}_1$. Note that $\tilde{b}_{11,i}, i \neq 1$, and $\tilde{b}_{12,j}$ appear only in the first column and the $(M+1)$st column, respectively and $\tilde{h}_{2i}$ appears only in the ith column, $2 \leq i \leq M$. Therefore, the cofactor C_{1j}'s cannot be canceled out in (4.8) and $|\tilde{\Gamma}_1|$ cannot be a zero-polynomial.

Since $|\tilde{\Gamma}_1|$ is not a zero-polynomial, it can be easily seen that $|\Gamma_1|$ cannot be a zero-polynomial. In fact, the set V of the root of $|\Gamma_1| = 0$ is an affine variety. Since $|\Gamma_1|$ is a non-zero polynomial, the dimension of this variety is lower than that of the continuous space U with $\tilde{\mathbf{b}}_{11}, \tilde{\mathbf{b}}_{12}, \mathbf{b}'_{11}$, h_{ji}'s, h_{ri}'s, and h_{jr}'s [39]. Therefore, V does not have volume in U.

Finally, since the volume of V is zero and all random variables are continuous one, we have

$$Pr(|\Gamma_1| = 0) = \int_V f(\tilde{\mathbf{b}}_{11}, \tilde{\mathbf{b}}_{12}, \mathbf{b}'_{11}, h_{ji}\text{'s}, h_{ri}\text{'s}, h_{jr}\text{'s})\, dU = 0$$

where $f(\cdot)$ is the joint probability density function. Since $|\Gamma_1|$ is not zero with probability 1, Γ_1 has full rank with probability 1 and thus the perfect IA can be achieved.

For the receiver 2, it can be similarly proved that the $|\Gamma_2|$ is not zero with probability 1 and therefore, the proposed IA scheme with a full-duplex relay can achieve the $2M/(M+1)$ DoF.

4.4.2. IA Scheme with Two Half-Duplex Relays

By considering arbitrary time extension, the equivalent channel matrix between the transmitter i and the receiver j for this scheme is given as

$$\mathbf{H}_{ji} =$$
$$h_{ji} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ \frac{u(1)h_{r_1 i}h_{jr_1}}{h_{ji}} & 1 & 0 & 0 & 0 & \cdots & 0 \\ \frac{u(2)u(1)g_1 h_{r_1 i}h_{jr_2}}{h_{ji}} & \frac{u(2)h_{r_2 i}h_{jr_2}}{h_{ji}} & 1 & 0 & 0 & \cdots & \vdots \\ \frac{u(3)u(2)u(1)g_1 g_2 h_{r_1 i}h_{jr_1}}{h_{ji}} & \frac{u(3)u(2)g_2 h_{r_2 i}h_{jr_1}}{h_{ji}} & \frac{u(3)h_{r_1 i}h_{jr_1}}{h_{ji}} & 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & & \ddots & \ddots & 0 \\ & & \text{Last low} & & & & \end{bmatrix}. \tag{4.9}$$

When M is odd, the last low in (4.9) is given as

$$\frac{\prod_{k=1}^{M} u(k) g_1^{\frac{M-1}{2}} g_2^{\frac{M-1}{2}} h_{r_1 i}h_{jr_1}}{h_{ji}} \quad \frac{\prod_{k=2}^{M} u(k) g_1^{\frac{M-3}{2}} g_2^{\frac{M-1}{2}} h_{r_2 i}h_{jr_1}}{h_{ji}} \cdots \frac{u(M)u(M-1)g_2 h_{r_2 i}h_{jr_1}}{h_{ji}} \quad \frac{u(M)h_{r_1 i}h_{jr_1}}{h_{ji}} \quad 1.$$

When M is even, the last low in (4.9) is given as

$$\frac{\prod_{k=1}^{M} u(k) g_1^{\frac{M}{2}} g_2^{\frac{M-2}{2}} h_{r_2 i} h_{j r_2}}{h_{ji}} \; \frac{\prod_{k=2}^{M} u(k) g_1^{\frac{M-2}{2}} g_2^{\frac{M-2}{2}} h_{r_1 i} h_{j r_2}}{h_{ji}} \cdots \frac{u(M) u(M-1) g_1 h_{r_1 i} h_{j r_2}}{h_{ji}} \; \frac{u(M) h_{r_2 i} h_{j r_2}}{h_{ji}} \; 1.$$

For simplicity, we assume that $u(k) = 1$ for all k. Let $\mathbf{H}_{ji} = h_{ji}(\mathbf{I} - 1/h_{ji}\mathbf{G}_{ji})$, where $\mathbf{G}_{ji}$ is the matrix consisting of zero diagonal elements and the negative values of off-diagonal entries of $\mathbf{H}_{ji}$. Clearly, $\mathbf{G}_{ji}$ is a nilpotent matrix and thus we have

$$\mathbf{H}_{ji}^{-1} = \frac{1}{h_{ji}} \sum_{k=0}^{M} h_{ji}^{-k} \mathbf{G}_{ji}^{k}$$

and

$$\mathbf{H}_{1i}\mathbf{H}_{2i}^{-1} = \frac{h_{1i}}{h_{2i}}(\mathbf{I} - \frac{1}{h_{1i}}\mathbf{G}_{1i}) \sum_{k=0}^{M} h_{2i}^{-k} \mathbf{G}_{2i}^{k}.$$

The first k rows of $\mathbf{G}_{2i}^{k}$ are all zero and $\sum_{k=0}^{M} h_{ji}^{-k}\mathbf{G}_{ji}^{k}$ contains higher order terms as the row index increases. The received signal space Γ_1 for the receiver 1 is defined similar to (4.6). Then the ith column $\Gamma_{1i} = \mathbf{H}_{1i}\mathbf{H}_{2i}^{-1}\mathbf{b}'_{11}$ of Γ_1 contains the polynomials of higher degrees as the row index increases. Consider the highest order terms of each entry of Γ_1 and then, $\tilde{\Gamma}_1$ can be obtained similarly to (4.7). Similar to the full-duplex relay case, $|\Gamma_1|$ can be shown to be a non-zero polynomial and thus the variety that $|\Gamma_1| = 0$ has lower dimensions than the union space consisting of all variables in $|\Gamma_1| = 0$. Therefore, the probability that $|\Gamma_1| = 0$ is zero and Γ_1 has full rank with probability one.

This proof can also be applied to the receiver 2 and therefore, this scheme also achieves the maximum DoF $2M/(M+1)$ for the $M \times 2$ X channel. It is clear that the proposed IA scheme with two half-duplex relays only needs to store the current received signal instead of all the received signals as in the conventional full-duplex relay-aided IA scheme [7].

In AF relay mode, the transmission power of the relay at time slot k is given as

$$P_{\hat{r}_k} = |u(k)|^2 \left(\sum_{i=1}^{M} |h_{\hat{r}_k i}|^2 |x_i(k)|^2 + N \right)$$

where N is the power of noise at the relay and it is assumed that $N = 1$. To keep the relay power stable on average for the proposed IA scheme with two half-duplex relays, $u(k)$ can be determined as

$$u(k) = \frac{1}{\sqrt{\sum_{i=1}^{M} |h_{r_1 i}|^2 + 1/\rho}} \text{ for } k = 1$$
$$u(k) = \frac{1}{\sqrt{\sum_{i=1}^{M} |h_{\hat{r}_k i}|^2 + |g_{\tilde{r}_k}|^2 + 1/\rho}} \text{ for } k > 1$$

where ρ denotes the transmit SNR at the transmitters and relays. $\hat{r}_k$ denotes r_1 and r_2 when k is odd and even, respectively and $\tilde{r}_k$ is 2 when $\hat{r}_k$ is r_1 and 1 when $\hat{r}_k$ is r_2. Then, for $E(|x|^2) = \rho$, it can be seen that $E(P_{\hat{r}_k}) = \rho$. It is clear that $u(k)$ increases as $|g_{\tilde{r}_k}|$ goes to 0 for $k > 1$, and the matrix $\mathbf{H}_{ji}$ in (4.9) has the similar form as that of the IA scheme with a full-duplex relay. Therefore, the perfect IA can be achieved in this case.

However, if $|g_{\tilde{r}_k}|$ goes to infinity, only the elements of the first column and diagonal elements $\mathbf{H}_{ji}$ in (4.9) are nonzero. In this case, the perfect IA is not feasible for even 2×2 X channel. To overcome this problem, the proposed IA scheme with two half-duplex relays should be modified.

Actually, the channel capacity between two half-duplex relays goes infinity in this case and thus each relay can send its information by using very small portion of bandwidth or time. This implies that two relays can share their information with almost no time delay and two half-duplex relays can operate like a full-duplex relay. Therefore, the proposed IA scheme with two half-duplex relays can behave like the proposed IA scheme with a full-duplex relay which can achieve the perfect IA.

4.5. Achievability of the Proposed IA Schemes for the $2 \times M$ X Channel

In this section, by using the reciprocity of the $2 \times M$ channel, it is shown that the perfect IA for the $2 \times M$ X channel is also possible for the proposed IA schemes and $2M/(M+1)$ DoF can be achieved [6]. The relation of the original $2 \times M$ X channel and its reciprocal $M \times 2$ X channel are explained in Fig. 4.5 [6].

Let $\mathbf{H}_{ji}$ and $\tilde{\mathbf{H}}_{ij}$ denote the $(M+1) \times (M+1)$ channel matrices between the transmitter i and the receiver j for the original $2 \times M$ X channel and between the transmitter j and the receiver i for its reciprocal $M \times 2$ X channel in Fig. 4.5, respectively. Then, we have the relation $\mathbf{H}_{ji} = \tilde{\mathbf{H}}_{ij}^{\dagger}$ [6]. $\mathbf{z}_{ji}$ denotes the zero-forcing vector for the beamforming vector $\mathbf{b}_{ji}$ at the receiver j in the original $2 \times M$ X channel and $\mathbf{z}_{ji}^{r}$ denotes the zero-forcing vector for the beamforming vector $\mathbf{b}_{ji}^{r}$ in the reciprocal $M \times 2$ X channel.

From Fig. 4.5, it is easy to check that the zero-forcing vectors for the original channel can be used as the beamforming vectors for the reciprocal

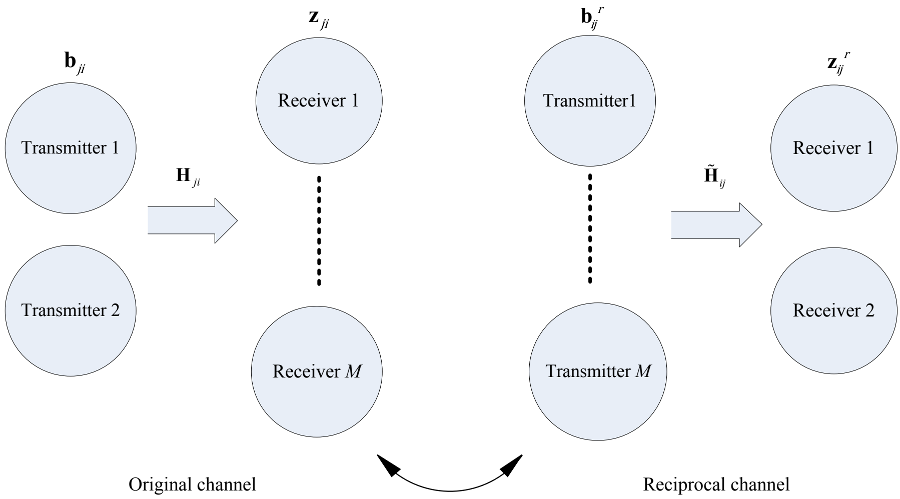

Figure 4.5: Reciprocal relation between $2 \times M$ and $M \times 2$ X channels.

channel and the beamforming vectors for the original channel can be used as the zero-forcing vectors for the reciprocal channel [6]. Therefore, if the perfect IA is possible for the reciprocal channel, it is also possible for the original channel.

In this section, it is assumed that the original channel is the $2 \times M$ X channel with relay and the reciprocal channel is the $M \times 2$ X channel with relay and $M + 1$ symbol extension is used. The system models are given as follows by considering a full-duplex relay and two half-duplex relays:

- The case of one full-duplex relay:

  The received signals at the relay and the receiver for the original channel are given as

$$y_r(k) = \sum_{i=1}^{2} h_{ri} x_i(k) + n_r(k)$$

$$y_j(k) = \sum_{i=1}^{2} h_{ji} x_i(k) + u(k) h_{jr} y_r(k-1) + n_j(k),\ \ j = 1, ..., M$$

where $k = 0, ..., M$ and $u(k)$ is the relay gain with $u(0) = 0$.

- The case of two half-duplex relays:

  For the original channel, the received signals at the relays are given as

$$y_{r_1}(k) = \sum_{i=1}^{2} h_{r_1 i} x_i(k) + g_2 u(k) y_{r_2}(k-1) + n_{r_1}(k),$$

$$k = 0, 2, 4, ... \text{ at relay } 1$$

$$y_{r_2}(k) = \sum_{i=1}^{2} h_{r_2 i} x_i(k) + g_1 u(k) y_{r_1}(k-1) + n_{r_2}(k),$$

$$k = 1, 3, 5, ... \text{ at relay } 2$$

  and the received signals at the receivers are also given as

$$y_j(k) = \begin{cases} \sum_{i=1}^{2} h_{ji} x_i(k) + u(k) h_{jr_2} y_{r_2}(k-1) + n_j(k), k = 0, 2, 4, ... \\ \sum_{i=1}^{2} h_{ji} x_i(k) + u(k) h_{jr_1} y_{r_1}(k-1) + n_j(k), k = 1, 3, 5, ... \end{cases}$$

  where $u(0) = 0$, $j = 1, ..., M$, and $k = 0, ..., M$.

For the original channel, the channel matrices with a full-duplex relay and two half-duplex relays are identical to $\mathbf{H}_{ji}$ in (4.5) and (4.9), respectively. Therefore, the channel matrices for the reciprocal channels $\tilde{\mathbf{H}}_{ij} = \mathbf{H}^*_{ji}$ become upper triangular matrices.

To verify the achievability of the proposed IA schemes for the $2 \times M$ X channel, we can use the results in Section 4.3 for the $M \times 2$ X channel by replacing the channel matrix with the upper triangular matrix $\tilde{\mathbf{H}}_{ji}$. Similar to the proof in Section 4.4, it is easy to show that Γ_1 for the reciprocal $M \times 2$ X channel in Fig. 4.4 has full rank, that is, a perfect IA is achieved for the reciprocal $M \times 2$ X channel. Therefore, it is clear that by setting $\mathbf{b}_{ji} = \mathbf{z}_{ij}^{r*}$, a perfect IA for the original $2 \times M$ channel is achieved by the IA schemes in [6].

4.6. Numerical Analysis

In this section, some numerical results are given to compare the proposed schemes with the relay-aided IA scheme in [7]. All channel coefficients and noises at the relays and receivers are assumed as complex Gaussian random variables distributed as $C(0,1)$. To satisfy the relay power constraint, the relay gain $u(k)$ is given as

$$u(k) = \begin{cases} \frac{1}{\sqrt{\sum_{i=1}^{M}|h_{ri}|^2+1/\rho}}, \text{ for the IA scheme with a full-duplex R} \\ \frac{1}{\sqrt{\sum_{i=1}^{M}|h_{r_1 i}|^2+1/\rho}}, k=1, \\ \qquad\qquad \text{for the IA scheme with two half-duplex R's} \\ \frac{1}{\sqrt{\sum_{i=1}^{M}|h_{\hat{r}_k i}|^2+|g_{\tilde{r}_k}|^2+1/\rho}}, k \neq 1, \\ \qquad\qquad \text{for the IA scheme with two half-duplex R's} \\ \frac{1}{\sqrt{n\left(\sum_{i=1}^{M}|h_{ri}|^2+1/\rho\right)}}, \text{ for the IA scheme in [7]} \end{cases} \tag{4.10}$$

where $k = 0, 1, ..., M$, with $u(0) = 0$. By setting the amplification gain of the relay as (4.10), the relay can keep its power stable on average. Actually, in [7], the amplification gain is set in a different way from (4.10). Since the relay stores all the received signals in $M+1$ time slots, the relay can amplify for each received signal by using different gain. Thus, optimization of IA schemes in [7] is possible. However, it is beyond the scope of this chapter and thus, the relay gain for the IA schemes in [7] is set as (4.10) for simplicity. Also, the zero-forcing vectors are obtained by using the method in [40].

Fig. 4.6 shows the average sum rate of the receiver 1 for various IA schemes with $M = 2, 3$, where 'Proposed IA (full)' denotes the proposed IA scheme with a full duplex relay and 'Proposed IA (half)' denotes the proposed IA scheme with two half-duplex relays. From Fig. 4.6, it can be seen that the sum rates of the proposed schemes are close to that in [7] and their slopes are also similar. Theoretically, for $M = 2$, each scheme can achieve DoF $2M/(M+1) = 4/3$ and thus, the DoF at the receiver 1 is $2/3$, where DoF d is defined as

$$\text{sum rate} \approx d \log_2(\text{SNR})$$

when SNR goes to infinity. In Fig. 4.6, we do not normalize the DoF by the time extension and if DoF 2 and DoF 3 are achieved at the receiver 1, it implies that the theoretical maximum DoF is achieved. In Fig. 4.6, the increase rate of the sum rate per one interval becomes close to 3.3 after about 25 dB SNR for $M = 2$, that is,

$$2\left(\log_2 \text{SNR}_2 - \log_2 \text{SNR}_1\right) = 2\log_2 10^{\Delta} \approx 3.32$$

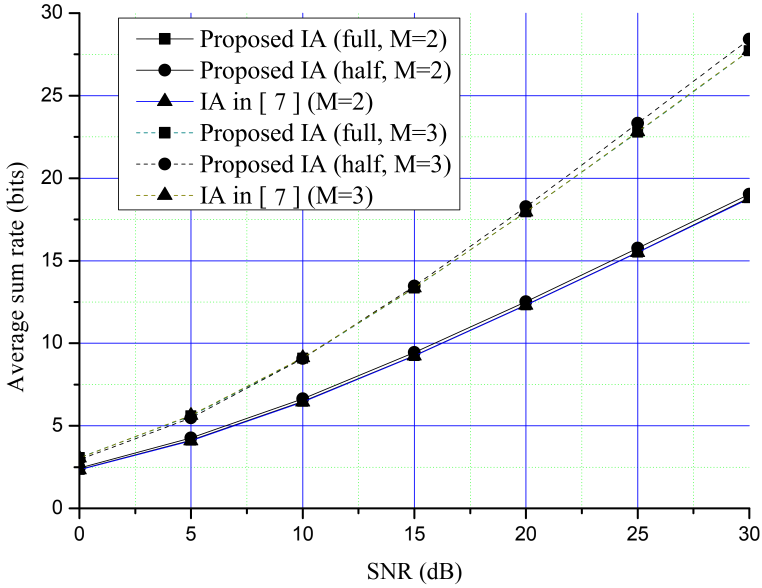

Figure 4.6: Average sum rate at the receiver 1 for various relay-aided IA schemes.

where $\Delta = \log_{10} \mathrm{SNR}_2/\mathrm{SNR}_1 = 0.5$ corresponds to the interval of the horizontal axis, 5dB SNR. Therefore, it is confirmed that the proposed schemes achieve the maximum DoF of $M \times 2$ X channel.

For $M = 3$, it can also be seen that the theoretical DoF is achieved by the proposed schemes.

In fact, the proposed IA scheme with two half-duplex relays shows a little better performance than the others. In the proposed IA scheme with two half-duplex relays, the receivers combine the signals from two independent channels due to two relays. It can be thought that the diversity effect is obtained a little and it provides a little better performance than when only one channel is used. However, there is a disadvantage that more CSI is needed at the transmitters and the receivers.

4.7. Conclusion

In this chapter, two relay-aided IA schemes are proposed for the quasi-static $M \times 2$ X channel by using a full-duplex relay and two half-duplex relays. It is proved that the proposed schemes can achieve the perfect IA and therefore, the maximum DoF $2M/(M+1)$ is also achieved. Through the numerical analysis, it is also shown that the proposed schemes can achieve the sum rate close to that of the previous relay-aided IA scheme. Therefore, the proposed IA schemes can be a good practical alternative schemes for the conventional relay-aided IA scheme. Especially, the second proposed scheme replaces the full-duplex relay with the half-duplex relay for IA and thus, it is more practical from the viewpoint of implementation.

As the further work, we can consider the IA scheme with one half-duplex relay and extension for the general X channel. In addition, the research on the combination of IA and diversity technique can also be a good topic in order to obtain high throughput and reliability simultaneously.

Chapter 5. Selection Diversity on the Interference Alignment for MIMO Interference Channels

5.1. Introduction

Recently, the IA has attracted great attention for interference management. Especially, for K-user interference channel and $M \times N$ X channel, IA schemes are proposed in [5, 6], which achieve the maximum DoF asymptotically. From the viewpoint of the throughput, DoF is a good measure for the spectral efficiency. However, the reliability is also important factor for wireless communications and the diversity order is the best-known measure for the reliability. Until now, the research on IA has been focused on DoF and thus there are only a few results related to diversity order of IA [41, 42].

In [41, 42], by using STBCs, the error performance is improved greatly for 2×2 X channel. In these results, each transmitter uses only the CSI for the connected links to oneself.

In fact, in many IA schemes, it is assumed that the global CSI is available at the transmitter and receiver and thus it is easily conjectured that selection scheme for beamforming matrices is more efficient than STBCs from the viewpoint of throughput. Therefore, in this chapter, we consider the selection scheme of beamforming matrices for IA.

In P2P MIMO communication systems, various selection schemes have been studied such as transmit antenna selection or equivalent path selection based on SVD. Since SNR at the receiver is the most critical factor, most of selection schemes for P2P MIMO communication systems are based on SNR at the receiver. However, these selection schemes are based on the zero-forcing (linear) receiver which brings out the degradation of error performance.

In general, the complexity of ML-decoding increases proportional to $\mathcal{M}^{\mathrm{DoF}}$, where $\mathcal{M}$ is the size of the signal constellation. However, its error performance is superior to that of the zero-forcing. Therefore, in this chapter, we consider two-stage decoding which consists of the zero-forcing and ML-decoding. In the two-stage decoding, the interference signal is removed by zero-forcing and then the desired signal is decoded by ML-decoding.

5.2. Characteristic Function of Multivariate Rayleigh Random Variables

Usually, the characteristic function of multivariate Rayleigh random variables is used for the analysis on the error probability. Therefore, in this section, we introduce the results in [45].

Consider $L \times 1$ complex Gaussian random vector $\mathbf{X}$ given by

$$\mathbf{X} = \mathbf{X}_c + j\mathbf{X}_s$$

$$= \begin{bmatrix} X_{c_1} \\ \vdots \\ X_{c_L} \end{bmatrix} + j \begin{bmatrix} X_{s_1} \\ \vdots \\ X_{s_L} \end{bmatrix} \tag{5.1}$$

with covariance matrix K_{cc} and K_{ss} and cross-covariance matrix K_{cs} such that $E[X_{c_i} X_{s_i}] = (K_{cs})_{ii} = 0$.

Let $\gamma_i = |X_{c_i} + jX_{s_i}|^2$ and $\gamma = (\gamma_i, \cdots, \gamma_L)^T$. Then the characteristic function $\Psi_\gamma(j\omega)$ of γ is given as [45]

$$\begin{aligned}
&\Psi_\gamma(j\omega) = E[\exp(j\omega\gamma)\\
&= (\det(I_L - 2j\mathrm{diag}(\omega)K_{ss}))^{-\frac{1}{2}}\\
&\times (\det([I_L - 2j\mathrm{diag}(\omega)K_{cc}] + 4\mathrm{diag}(\omega)K_{cs}[I_L - 2j\mathrm{diag}(\omega)K_{ss}]^{-1}\\
&\times \mathrm{diag}(\omega)K_{cs}^T)^{-\frac{1}{2}}.
\end{aligned} \tag{5.2}$$

If **X** is a circularly symmetric complex Gaussian random vector, that is

$$K_{cc} = K_{ss}, K_{cs}^T = -K_{cs},$$

(5.2) is simplified as

$$\begin{aligned}
\Psi_\gamma(j\omega) = \det(I_L - 2j\mathrm{diag}(\omega)[K_{cc} + jK_{cs}] \times \det(I_L - 2j\mathrm{diag}(\omega)[K_{cc} - jK_{cs}]\\
\det(I_L - j\mathrm{diag}(\omega)K)^{-\frac{1}{2}} \times \det(I_L - j\mathrm{diag}(\omega)K^\dagger)^{-\frac{1}{2}}
\end{aligned} \tag{5.3}$$

where K is the covariance matrix of **X**.

In general, for analysis on the error performance, (5.2) and (5.3) are important formulas because the error probability is given as a Q function, which is upper-bounded by the exponential function. Let $\omega = j(\rho, \rho, \cdots, \rho)^T$

and then (5.2) is given as

$$\Psi_\gamma(-\rho) = (\det(I_L + 2\rho K_{ss}))^{-\frac{1}{2}} \\ \times \left(\det([I_L + 2\rho K_{cc}] + 4\rho^2 K_{cs}[I_L + 2\rho K_{ss}]^{-1} K_{cs}^T\right)^{-\frac{1}{2}}$$

and (5.3) is also given as

$$\Psi_\gamma(-\rho) = \det(I_L + \rho K)^{-\frac{1}{2}} \times \det(I_L + \rho K^\dagger)^{-\frac{1}{2}}$$

In high SNR region, where $\rho \to \infty$, we have

$$\Psi_\gamma(-\rho) \approx (2\rho)^{-\frac{\mathrm{rank}(K_{ss})+\mathrm{rank}(K_{cc}+K_{cs}K_{ss}^{-1}K_{cs}^T)}{2}} \prod \lambda_{K_{ss}} \prod \lambda_{K_{cc}+K_{cs}K_{ss}^{-1}K_{cs}^T}$$

where λ_* denotes the eigenvalues of the matrix. If there is no deterministic relation among the element of $\mathbf{X}$, K is a full-rank matrix.

For (5.3), we have

$$\Psi_\gamma(-\rho) = \rho^{-\mathrm{rank}(K)} |\prod \lambda_K|^2. \tag{5.4}$$

From this result, the following lemma can be obtained.

Lemma 5.1. Consider that the received SNR is expressed as $\gamma = \rho|\mathbf{X}|^2$, where $\mathbf{X}$ is a circularly symmetric Gaussian random vector. The diversity order of the decoder is the number of the random variables with non-deterministic relation in $\mathbf{X}$.

5.3. Combining IA and Selection Schemes for 3-User MIMO Interference Channel

In this section, the system model for 3-user MIMO interference channel and IA is explained. And then, the proposed scheme, the selection scheme of beamforming matrices for IA, is introduced.

5.3.1. System Model and IA for 3-User MIMO Interference Channel

Here is a system model for the 3-user interference channel with M antennas.

- For even M, we have

$$\mathbf{Y}_{e,k} = \sqrt{\rho}\sum_{i=1}^{3}\mathbf{H}_{ki}\mathbf{V}_i\mathbf{d}_k + \mathbf{n}_k$$

  where $\mathbf{H}_{ki}$ and $\mathbf{V}_i$ denote $M \times M$ channel matrix between transmitter i and receiver k and $M \times \frac{M}{2}$ beamforming matrix of transmitter i, respectively. $\mathbf{d}_k$ is an $\frac{M}{2} \times 1$ data stream vector and ρ is the parameter linearly proportional to the average transmit SNR. It is assumed that all channel coefficients are i.i.d with the Rayleigh distribution.

- For odd M, we have

$$\mathbf{Y}_{o,k} = \sqrt{\rho}\sum_{i=1}^{3}\mathbf{H}_{o,ki}\mathbf{V}_i\mathbf{d}_k + \mathbf{n}_k$$

where $\mathbf{H}_{o,ki}$ is given as

$$\mathbf{H}_{o,ki} = \begin{bmatrix} \mathbf{H}_{ki} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{ki} \end{bmatrix}$$

and $\mathbf{V}_i$ denotes $2M \times M$ beamforming matrix, respectively. $\mathbf{d}_k$ is an $M \times 1$ data stream vector.

We can generate the beamforming matrices for IA as follows. From the result in [5], the beamforming matrices for IA are obtained by the following relation.

$$\begin{aligned} \mathrm{span}(\mathbf{H}_{12}\mathbf{V}_2) &= \mathrm{span}(\mathbf{H}_{13}\mathbf{V}_3) \\ \mathbf{H}_{21}\mathbf{V}_1 &= \mathbf{H}_{23}\mathbf{V}_3 \\ \mathbf{H}_{31}\mathbf{V}_1 &= \mathbf{H}_{32}\mathbf{V}_2. \end{aligned} \tag{5.5}$$

It is rewritten as

$$\begin{aligned} \mathrm{span}(\mathbf{V}_1) &= \mathrm{span}(\mathbf{H}_{31}^{-1}\mathbf{H}_{32}\mathbf{H}_{12}^{-1}\mathbf{H}_{13}\mathbf{H}_{23}^{-1}\mathbf{H}_{21}\mathbf{V}_1) \\ \mathbf{V}_2 &= \mathbf{H}_{32}^{-1}\mathbf{H}_{31}\mathbf{V}_1 \\ \mathbf{V}_{\mathbf{3}} &= \mathbf{H}_{23}^{-1}\mathbf{H}_{21}\mathbf{V}_1. \end{aligned} \tag{5.6}$$

Therefore, $\mathbf{V}_{\mathbf{1}}$ consists of the eigenvectors of $\mathbf{H}_{31}^{-1}\mathbf{H}_{32}\mathbf{H}_{12}^{-1}\mathbf{H}_{13}\mathbf{H}_{23}^{-1}\mathbf{H}_{21}(=\mathbf{A})$ and $\mathbf{V}_2$ and $\mathbf{V}_3$ can be obtained from (5.6).

- For even M, we have

$$\mathbf{V}_1 = \left[\mathbf{u}_1\mathbf{u}_2 \cdots \mathbf{u}_{\frac{M}{2}}\right] \tag{5.7}$$

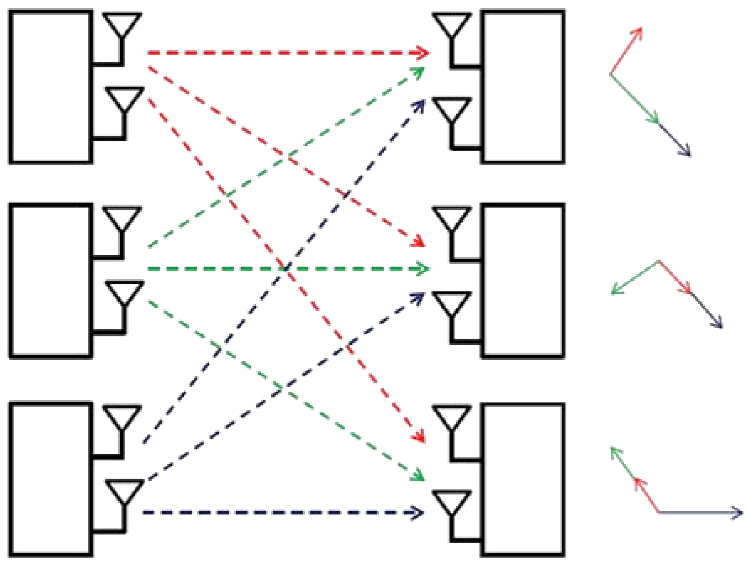

Figure 5.1: IA for 3-user interference channel.

where $\mathbf{u}_i$ is an eigenvector of $\mathbf{A}$.

- For odd M, we have

$$\mathbf{V}_1 = \begin{bmatrix} \mathbf{u}_1 & 0 & \mathbf{u}_3 & 0 \cdots & 0 & \mathbf{u}_M \\ 0 & \mathbf{u}_2 & 0 & \mathbf{u}_4 \cdots & \mathbf{u}_{M-1} & 0 \end{bmatrix}. \tag{5.8}$$

In this way, at each receiver, interference signal vectors are aligned as described in Fig. 5.1. In fact, for odd M, all eigenvectors of $\mathbf{A}$ should be used as in (5.8) and thus the selection of beamforming matrices is complicated and IA should be modified for selection. Therefore, for simplicity, we consider the case that M is even in this chapter.

5.3.2. Orthogonalization of Beamforming Matrices

In general, the orthogonalization of transmit signals can achieve the improvement of performance. Equation (5.6) guarantees the perfect IA but does not guarantee the orthogonalization between beamforming vectors at each transmitter. Therefore, we should modify the design of beamforming matrices.

In fact, $\mathbf{V}_1$ is generated by the eigenvectors of $\mathbf{A}$ and thus its column vectors are orthogonal each other. For $\mathbf{V}_2$ and $\mathbf{V}_3$, we can design them, by using the following relation [46].

$$\text{span}(\mathbf{V}_2) = \text{span}(\mathbf{H}_{32}^{-1}\mathbf{H}_{31}\mathbf{V}_1) = \text{span}(\mathbf{Q}_2\mathbf{R}_2) = \text{span}(\mathbf{Q}_2)$$
$$\text{span}(\mathbf{V}_{\mathbf{3}}) = \text{span}(\mathbf{H}_{23}^{-1}\mathbf{H}_{21}\mathbf{V}_1) = \text{span}(\mathbf{Q}_3\mathbf{R}_3) = \text{span}(\mathbf{Q}_3)$$

where $\mathbf{Q}_i\mathbf{R}_i$ is obtained by QR factorization and $\mathbf{Q}_i$ consists of orthonormal column vectors. Therefore, we can use $\mathbf{Q}_i$ as a beamforming matrix for the transmitter i in order to improve the performance.

5.3.3. Selection of Beamforming Matrices

In [5], each transmitter can send $M/2$ data streams and each receiver can detect them without interference. In general, each receiver decodes the desired signal by only zero-forcing because of its complexity. However, the zero-forcing can make the error performance worse. Therefore, in this section, we consider two-stage decoding procedure which consists of zero-forcing for the interference signal and ML decoding for the desired signal. This decoding procedure is described in Fig. 5.2.

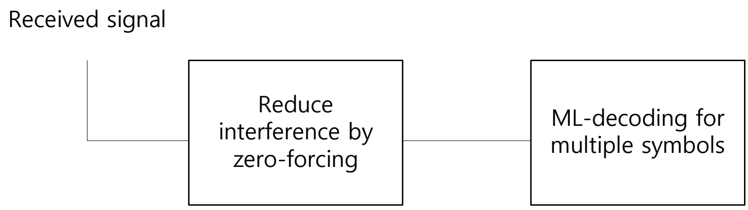

Figure 5.2: Decoding procedure at each receiver.

In MIMO communication system, given a channel matrix $\mathbf{H}$, the PEP is given as

$$P_{\text{PEP}} = \Pr(\mathbf{x} \to \hat{\mathbf{x}}|\mathbf{H}) = Q(|\mathbf{H}(\mathbf{x} - \hat{\mathbf{x}})|).$$

Considering M-QAM, the SER can be obtained approximately by union bound of PEP and thus lowering PEP is related to lowering SER directly. Therefore, from the viewpoint of PEP, we can select the eigenvectors for $\mathbf{V}_1$ by using the following criterion

$$\mathbf{V}_{1,\text{sel}} = \arg \max_{\mathbf{V}_1 \in \text{eig}(\mathbf{A}),\mathbf{d}} \min(|\mathbf{R}_1^{\dagger}\mathbf{H}_{11}\mathbf{V}_1\mathbf{d}|, |\mathbf{R}_2^{\dagger}\mathbf{H}_{22}\mathbf{V}_2\mathbf{d}|, |\mathbf{R}_3^{\dagger}\mathbf{H}_{33}\mathbf{V}_3\mathbf{d}|) \tag{5.9}$$

where $\mathbf{d} = \mathbf{x} - \hat{\mathbf{x}}$. $\mathbf{d}$ can be various vectors for M-QAM because there are various error patterns.

5.4. Diversity Analysis

In this section, the diversity order of the proposed scheme is derived for $M = 2$. From the PEP when the selection is applied, the following relation is given as

$$E_{\mathbf{H}_{11},\mathbf{H}_{22},\mathbf{H}_{33}} Pr(\mathbf{x} \to \hat{\mathbf{x}}|\mathbf{A}, \mathbf{H}_{11}, \mathbf{H}_{22}, \mathbf{H}_{33})$$

$$
\begin{aligned}
&= \frac{1}{3}E_{\mathbf{H}_{11},\mathbf{H}_{22},\mathbf{H}_{33}}(Pr(\mathbf{x}\rightarrow\hat{\mathbf{x}}|\mathbf{A},\mathbf{H}_{11}) + \Pr(\mathbf{x}\rightarrow\hat{\mathbf{x}}|\mathbf{A},\mathbf{H}_{22})\\
&+ Pr(\mathbf{x}\rightarrow\hat{\mathbf{x}}|\mathbf{A},\mathbf{H}_{33}))\\
&\leq \frac{1}{3}E_{\mathbf{H}_{11},\mathbf{H}_{22},\mathbf{H}_{33}}\sum_{i=1,2,3} Q(\sqrt{\rho}|\mathbf{R}_{i,\mathrm{sel}}^{\dagger}\mathbf{H}_{ii}\mathbf{V}_{i,\mathrm{sel}}\mathbf{d}|)\\
&\leq E_{\mathbf{H}_{11},\mathbf{H}_{22},\mathbf{H}_{33}}Q(\sqrt{\rho}\min_{i=1,2,3}(|\mathbf{R}_{i,\mathrm{sel}}^{\dagger}\mathbf{H}_{ii}\mathbf{V}_{i,\mathrm{sel}}\mathbf{d}|))\\
&= E_{\mathbf{H}_{11},\mathbf{H}_{22},\mathbf{H}_{33}}Q(\sqrt{\rho}\max(\min_{i_1=1,2,3}(|\mathbf{R}_{i_1,1}^{\dagger}\mathbf{H}_{i_1 i_1}\mathbf{V}_{i_1,1}\mathbf{d}|),\\
&\qquad\qquad \min_{i_2=1,2,3}(|\mathbf{R}_{i_2,2}^{\dagger}\mathbf{H}_{i_2 i_2}\mathbf{V}_{i_2,2}\mathbf{d}|))\\
&\leq E_{\mathbf{H}_{11},\mathbf{H}_{22},\mathbf{H}_{33}}\exp(-\frac{\rho}{2}\max(\min_{i_1=1,2,3}(|\mathbf{R}_{i_1,1}^{\dagger}\mathbf{H}_{i_1 i_1}\mathbf{V}_{i_1,1}\mathbf{d}|^2),\\
&\qquad\qquad \min_{i_2=1,2,3}(|\mathbf{R}_{i,2}^{\dagger}\mathbf{H}_{ii}\mathbf{V}_{i,2}\mathbf{d}|^2))\\
&\leq E_{\mathbf{H}_{11},\mathbf{H}_{22},\mathbf{H}_{33}}\exp(-\frac{\rho}{2}(\min_{i_1=1,2,3}(|\mathbf{R}_{i_1,1}^{\dagger}\mathbf{H}_{i_1 i_1}\mathbf{V}_{i_!,1}\mathbf{d}|^2)\\
&\qquad\qquad + \min_{i_2=1,2,3}(|\mathbf{R}_{i_2,2}^{\dagger}\mathbf{H}_{i_2 i_2}\mathbf{V}_{i,2}\mathbf{d}|^2))\\
&\leq \max_{(i_1,i_2)\in(1,2,3)^2}\left\{E_{\mathbf{H}_{11},\mathbf{H}_{22},\mathbf{H}_{33}}\exp(-\frac{\rho}{2}(|\mathbf{R}_{i_1,1}^{\dagger}\mathbf{H}_{i_1 i_1}\mathbf{V}_{i_1,1}\mathbf{d}|^2 + |\mathbf{R}_{i_2,2}^{\dagger}\mathbf{H}_{i_2 i_2}\mathbf{V}_{i_2,2}\mathbf{d}|^2)\right\}\\
&\leq \sum_{(i_1,i_2)\in(1,2,3)^2}\left\{E_{\mathbf{H}_{11},\mathbf{H}_{22},\mathbf{H}_{33}}\exp(-\frac{\rho}{2}(|\mathbf{R}_{i_1,1}^{\dagger}\mathbf{H}_{i_1 i_1}\mathbf{V}_{i_1,1}\mathbf{d}|^2 + |\mathbf{R}_{i_2,2}^{\dagger}\mathbf{H}_{i_2 i_2}\mathbf{V}_{i_2,2}\mathbf{d}|^2)\right\}
\end{aligned}
$$

where $\mathbf{R}_{i_j,j}$ and $\mathbf{V}_{i_j,j}$ denote the zero-forcing and beamforming matrices for j, respectively. $\mathbf{R}_{i,\mathrm{sel}}$ and $\mathbf{V}_{i,\mathrm{sel}}$ denote the selected zero-forcing and beamforming matrices. In this case, $\mathbf{R}_{i_j,j}$ and $\mathbf{V}_{i_j,j}$ are 2×1 vectors and $\mathbf{d}$ is a scalar.

Let $h_{i_1,1} = \mathbf{R}_{i_1,1}^{\dagger}\mathbf{H}_{i_1 i_1}\mathbf{V}_{i_1,1}\mathbf{d}$ and $h_{i_2,2} = \mathbf{R}_{i_2,2}^{\dagger}\mathbf{H}_{i_2 i_2}\mathbf{V}_{i_2,2}\mathbf{d}$ and let $\gamma = |h_{i_1,1}|^2 + |h_{i_2,2}|^2$, where $i_1, i_2 \in \{1,2,3\}$. If $i_1 \neq i_2$, γ is the sum of two independent exponential random variables because all channel matrices are generated independently. If $i_1 = i_2$, γ is the sum of two correlated exponential random variables. In this case, we obtain the following covariance matrices

$$K_{ss} = \begin{bmatrix} E(\mathcal{I}(h_{i_1,1})\mathcal{I}(h_{i_1,1}) & E(\mathcal{I}(h_{i_1,1})\mathcal{I}(h_{i_1,2}) \\ E(\mathcal{I}(h_{i_1,2})\mathcal{I}(h_{i_1,1}) & E(\mathcal{I}(h_{i_1,2})\mathcal{I}(h_{i_1,2}) \end{bmatrix}$$

$$K_{cc} = \begin{bmatrix} E(\mathcal{R}(h_{i_1,1})\mathcal{R}(h_{i_1,1}) & E(\mathcal{R}(h_{i_1,1})\mathcal{R}(h_{i_1,2}) \\ E(\mathcal{R}(h_{i_1,2})\mathcal{R}(h_{i_1,1}) & E(\mathcal{R}(h_{i_1,2})\mathcal{R}(h_{i_1,2}) \end{bmatrix}$$

$$K_{cs} = \begin{bmatrix} E(\mathcal{R}(h_{i_1,1})\mathcal{I}(h_{i_1,1}) & E(\mathcal{R}(h_{i_1,1})\mathcal{I}(h_{i_1,2}) \\ E(\mathcal{R}(h_{i_1,2})\mathcal{I}(h_{i_1,1}) & E(\mathcal{R}(h_{i_1,2})\mathcal{I}(h_{i_1,2}) \end{bmatrix}.$$

For simplicity, let $h_{i_1,k} = \mathbf{R}_k\mathbf{H}\mathbf{V}_k$, and then, we have

$$\mathcal{R}(h_{i_1,k}) = \mathbf{R}_{k,q}^T\mathbf{H}_q\mathbf{V}_{k,q} - \mathbf{R}_{k,I}^T\mathbf{H}_I\mathbf{V}_{k,q} - \mathbf{R}_{k,q}^T\mathbf{H}_I\mathbf{V}_{k,I} - \mathbf{R}_{k,I}^T\mathbf{H}_q\mathbf{V}_{1,I}$$
$$\mathcal{I}(h_{i_1,k}) = \mathbf{R}_{k,q}^T\mathbf{H}_q\mathbf{V}_{k,I} - \mathbf{R}_{k,I}^T\mathbf{H}_I\mathbf{V}_{k,I} + \mathbf{R}_{k,q}^T\mathbf{H}_I\mathbf{V}_{k,q} + \mathbf{R}_{k,I}^T\mathbf{H}_q\mathbf{V}_{1,q}$$

where $()_q$ and $()_I$ denote the real and imaginary parts of the matrix, respectively. Therefore,

$$\begin{aligned}
&E(\mathcal{I}(h_{i_1,k})\mathcal{I}(h_{i_1,l})) \\
&= \mathbf{R}_{k,q}^T\mathbf{H}_q\mathbf{V}_{k,I}\mathbf{R}_{l,q}^T\mathbf{H}_q\mathbf{V}_{l,I} + \mathbf{R}_{k,q}^T\mathbf{H}_q\mathbf{V}_{k,I}\mathbf{R}_{l,I}^T\mathbf{H}_q\mathbf{V}_{l,q} \\
&+ \mathbf{R}_{k,I}^T\mathbf{H}_I\mathbf{V}_{k,I}\mathbf{R}_{l,I}^T\mathbf{H}_I\mathbf{V}_{l,I} - \mathbf{R}_{k,I}^T\mathbf{H}_I\mathbf{V}_{k,I}\mathbf{R}_{l,q}^T\mathbf{H}_I\mathbf{V}_{l,q} \\
&- \mathbf{R}_{k,q}^T\mathbf{H}_I\mathbf{V}_{k,q}\mathbf{R}_{l,I}^T\mathbf{H}_I\mathbf{V}_{l,I}^T + \mathbf{R}_{k,q}^T\mathbf{H}_I\mathbf{V}_{k,q}\mathbf{R}_{l,q}^T\mathbf{H}_I\mathbf{V}_{l,q} \\
&+ \mathbf{R}_{k,I}^T\mathbf{H}_q\mathbf{V}_{k,q}\mathbf{R}_{l,q}^T\mathbf{H}_q\mathbf{V}_{l,I} + \mathbf{R}_{k,I}^T\mathbf{H}_q\mathbf{V}_{k,q}\mathbf{R}_{l,I}^T\mathbf{H}_q\mathbf{V}_{l,q} \\
&E(\mathcal{R}(h_{i_1,k})\mathcal{R}(h_{i_1,l})) \\
&= \mathbf{R}_{k,q}^T\mathbf{H}_q\mathbf{V}_{k,q}\mathbf{R}_{l,q}^T\mathbf{H}_q\mathbf{V}_{l,q} - \mathbf{R}_{k,q}^T\mathbf{H}_q\mathbf{V}_{k,q}\mathbf{R}_{l,I}^T\mathbf{H}_q\mathbf{V}_{l,I}
\end{aligned}$$

$$+\mathbf{R}_{k,I}^T\mathbf{H}_I\mathbf{V}_{k,q}\mathbf{R}_{l,I}^T\mathbf{H}_I\mathbf{V}_{l,q}+\mathbf{R}_{k,I}^T\mathbf{H}_I\mathbf{V}_{k,q}\mathbf{R}_{l,q}^T\mathbf{H}_I\mathbf{V}_{l,I}$$

$$+\mathbf{R}_{k,q}^T\mathbf{H}_I\mathbf{V}_{k,I}\mathbf{R}_{l,I}^T\mathbf{H}_I\mathbf{V}_{l,q}^T+\mathbf{R}_{k,q}^T\mathbf{H}_I\mathbf{V}_{k,I}\mathbf{R}_{l,q}^T\mathbf{H}_I\mathbf{V}_{l,I}$$

$$-\mathbf{R}_{k,I}^T\mathbf{H}_q\mathbf{V}_{k,I}\mathbf{R}_{l,q}^T\mathbf{H}_q\mathbf{V}_{l,q}+\mathbf{R}_{k,I}^T\mathbf{H}_q\mathbf{V}_{k,I}\mathbf{R}_{l,I}^T\mathbf{H}_q\mathbf{V}_{l,I}$$

$$E(\mathcal{R}(h_{i_1,k})\mathcal{I}(h_{i_1,l}))$$

$$=\mathbf{R}_{k,q}^T\mathbf{H}_q\mathbf{V}_{k,q}\mathbf{R}_{l,q}^T\mathbf{H}_q\mathbf{V}_{l,I}+\mathbf{R}_{k,q}^T\mathbf{H}_q\mathbf{V}_{k,q}\mathbf{R}_{l,I}^T\mathbf{H}_q\mathbf{V}_{l,q}$$

$$+\mathbf{R}_{k,I}^T\mathbf{H}_I\mathbf{V}_{k,q}\mathbf{R}_{l,I}^T\mathbf{H}_I\mathbf{V}_{l,I}-\mathbf{R}_{k,I}^T\mathbf{H}_I\mathbf{V}_{k,q}\mathbf{R}_{l,q}^T\mathbf{H}_I\mathbf{V}_{l,q}$$

$$+\mathbf{R}_{k,q}^T\mathbf{H}_I\mathbf{V}_{k,I}\mathbf{R}_{l,I}^T\mathbf{H}_I\mathbf{V}_{l,I}^T-\mathbf{R}_{k,q}^T\mathbf{H}_I\mathbf{V}_{k,I}\mathbf{R}_{l,q}^T\mathbf{H}_I\mathbf{V}_{l,q}$$

$$-\mathbf{R}_{k,I}^T\mathbf{H}_q\mathbf{V}_{k,I}\mathbf{R}_{l,q}^T\mathbf{H}_q\mathbf{V}_{l,I}-\mathbf{R}_{k,I}^T\mathbf{H}_q\mathbf{V}_{k,I}\mathbf{R}_{l,I}^T\mathbf{H}_q\mathbf{V}_{l,q}$$

$$E(\mathcal{I}(h_{i_1,k})\mathcal{R}(h_{i_1,l}))$$

$$=\mathbf{R}_{k,q}^T\mathbf{H}_q\mathbf{V}_{k,I}\mathbf{R}_{l,q}^T\mathbf{H}_q\mathbf{V}_{l,q}-\mathbf{R}_{k,q}^T\mathbf{H}_q\mathbf{V}_{k,I}\mathbf{R}_{l,I}^T\mathbf{H}_q\mathbf{V}_{l,I}$$

$$+\mathbf{R}_{k,I}^T\mathbf{H}_I\mathbf{V}_{k,I}\mathbf{R}_{l,I}^T\mathbf{H}_I\mathbf{V}_{l,q}+\mathbf{R}_{k,I}^T\mathbf{H}_I\mathbf{V}_{k,I}\mathbf{R}_{l,q}^T\mathbf{H}_I\mathbf{V}_{l,I}$$

$$-\mathbf{R}_{k,q}^T\mathbf{H}_I\mathbf{V}_{k,q}\mathbf{R}_{l,I}^T\mathbf{H}_I\mathbf{V}_{l,q}^T-\mathbf{R}_{k,q}^T\mathbf{H}_I\mathbf{V}_{k,q}\mathbf{R}_{l,q}^T\mathbf{H}_I\mathbf{V}_{l,I}$$

$$+\mathbf{R}_{k,I}^T\mathbf{H}_q\mathbf{V}_{k,q}\mathbf{R}_{l,q}^T\mathbf{H}_q\mathbf{V}_{l,q}-\mathbf{R}_{k,I}^T\mathbf{H}_q\mathbf{V}_{k,q}\mathbf{R}_{l,I}^T\mathbf{H}_q\mathbf{V}_{l,I}. \tag{5.10}$$

Since $\mathbf{H}_q$ and $\mathbf{H}_I$ are i.i.d, $K_{ss} = K_{cc}$. From (5.10), it can be seen that $K_{cs} = E(\mathcal{R}(h_{i_1\mathrm{w},1})\mathcal{I}(h_{\mathrm{i}_1,2}))\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$. Therefore, $\mathbf{h}_{i_1,k}$'s are circularly symmetric joint Gaussian random variables.

Since two beamforming vectors and zero-forcing vectors are almost surely linearly independent and thus there is no deterministic relation between $h_{i_1,1}$ and $h_{i_1,2}$, which implies that $\mathrm{rank}(K_{h_{i_1,1},h_{i_1,2}}) = 2$ with probability one. Therefore, from Lemma 5.1, the diversity order is two in this case.

Let $|h_{i_1,1}|^2 = x_1$ and $|h_{i_2,2}|^2 = x_2$. For $i_1 \neq i_2$, since these two complex Gaussian random variables are independent each other, we have

$$
\begin{aligned}
& E_{\mathbf{H}_{11},\mathbf{H}_{22},\mathbf{H}_{33}}[\exp(-\rho\gamma)] \\
&= E_{\mathbf{H}_{11},\mathbf{H}_{22},\mathbf{H}_{33}}[\exp(-\rho x_1 - \rho x_2)] \\
&= E_{\mathbf{H}_{11},\mathbf{H}_{22},\mathbf{H}_{33}}[\exp(-\rho x_1)] \times E_{\mathbf{H}_{11}\mathbf{H}_{22},\mathbf{H}_{33}}[\exp(-\rho x_2))] \\
&= E_{\mathbf{H}_{i_1,i_1}}[\exp(-\rho x_1)] \times E_{\mathbf{H}_{i_2 i_2}}[\exp(-\rho x_2^2))] \\
&= \int_0^\infty \exp(-\rho x_1)\lambda_1 \exp\left(-\lambda_1 x_1\right) \exp(-\rho x_2)\lambda_2 \exp\left(-\lambda_2 x_2\right) \\
&= \frac{\lambda_1\lambda_2}{(\rho+\lambda_1)(\rho+\lambda_2)}
\end{aligned}
\tag{5.11}
$$

where λ_1 and λ_2 are the rate parameters of x_1 and x_2, respectively. As ρ increases, (5.11) becomes close to $\lambda_1\lambda_2/\rho^{-2}$ and the diversity order is 2. Therefore, by using the selection, diversity 2 can be achieved.

5.5. Simulation Results

In this section, some simulation results for the proposed scheme are presented. Each transmitter uses BPSK for each symbol and it is assumed that the distributions of all channel coefficients are identical.

In Fig 5.3, the performances of various schemes are compared for $M = 2, 4$. In fact, for $M = 2$, the two-stage decoding is the same as the zero-forcing decoding and the orthogonalization of beamforming matrices is not required because the desired symbol is one but for $M = 4$, two schemes become different.

It can be easily seen that the performance of two-stage decoding is

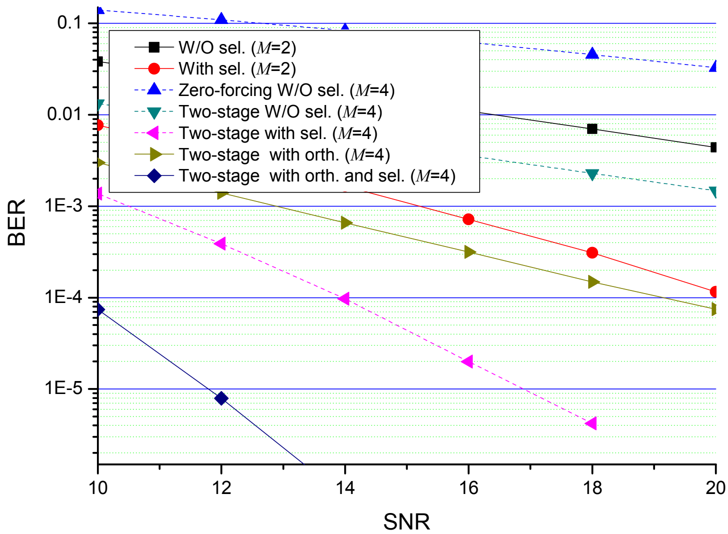

Figure 5.3: Performance comparison between the proposed scheme and the conventional IA.

superior to that of zero-forcing decoding and through the orthogonalization and selection of beamforming matrices, the error performance can be improved greatly. In fact, in the case that the only zero-forcing decoder is used, for decoding one desired symbol, the other desired symbols are treated like interference symbols and thus the desired signals can experience degradation. Therefore, for $M = 4$, the performance of the zero-forcing becomes worse than that for $M = 2$.

For $M = 2$, the diversity order of the proposed scheme becomes close to two as SNR increases and its performance is superior to that of IA without selection. This is natural result because the IA without selection achieves diversity order one.

Also, from the gap between performances for $M = 2$ and $M = 4$, it is expected that the diversity order of the two-stage decoding with the selection and orthogonalization of beamforming matrices becomes greater as the number of antennas increases.

5.6. Conclusions

In this chapter, a selection method of beamforming matrices for IA is proposed and its diversity order is analyzed for $M = 2$. For the error performance enhancement, two-stage decoding is considered which consists of the zero-forcing for mitigation of interference signals and the ML decoding for the desired signals.

By using the selection scheme of beamforming matrices and two-stage decoding, it is confirmed that IA can achieve more diversity order. Especially, it is proved that diversity order two can achieved by the proposed scheme for $M = 2$. And through the simulation, it is shown that the performance of the proposed scheme is superior to that of the conventional IA.

As a further work, for $M > 2$, we consider to analyze the exact diversity order. In addition, the proposed scheme can be easily extended to the other wireless communication environment where IA can be used. Therefore, the analysis on the selection diversity for the other channel models is considered as a good further work.

Chapter 6. Conclusions

In this dissertation, we derive the ROT for NDF protocol with SAS and OSTBCs. And then, through the diversity analysis, it is proved that the proposed relay on-off scheme achieves the maximum diversity order for NDF protocol with SAS.

In the second part of this dissertation, new relay IA schemes aided by relays are proposed for quasi-static $M \times 2$ X channel and it is confirmed that the proposed schemes can achieve the maximum DoF for $M \times 2$ X channel.

In the last part of this dissertation, by using a selection scheme of beamforming matrices for IA, it is shown that the reliability of IA can be enhanced.

In Chapter 2, MIMO, cooperative communications, and IA have been reviewed. The advantages of MIMO can be obtained by using the cooperative communications and STCs and IA can be implemented by using relays as virtual antennas for source to destination.

In Chapter 3, the ROT for NDF protocol with SAS and OSTBC is derived. The ROT is obtained to minimize the SER under M-QAM and it is proved that the relay on-off scheme with the ROT can achieve diversity order $2M_SM_D + \min(M_SM_R, M_RM_D)$, which is the maximum diversity order of NDF protocol with SAS. Through the simulation, it is confirmed that the ROT works well and its error performance is close to the optimal ROT which is obtained by extensive simulation.

In Chapter 4, we review the IA for X channel and propose new IA schemes aided by relays for $M \times 2$ channel. In the conventional IA scheme, it is assumed that the channel is time-varying and the perfect CSI is available at the transmitter. In fact, these two assumptions are contradictory. However, in the proposed schemes, the quasi-static channel is assumed and by using relays, it is proved that the maximum DoF for $M \times 2$ X channel can be achieved. Especially, it is shown that by using the two half-duplex relays, IA can also be implemented. Through the numerical analysis, it is confirmed that the proposed IA scheme with two half-duplex relays shows slightly better performance than the IA scheme with a full-duplex relay for $M \times 2$ X channel.

In Chapter 5, a selection scheme of beamforming matrices for IA in the 3-user interference channel is proposed and its diversity order is analyzed. In general, the research on the IA has been focused on the DoF which is the main measure of throughput. In wireless communications, the reliability is also important and thus the research on the diversity technique is also required for IA. Since IA is possible when the global CSI is available at the transmitters, the selection scheme for MIMO can be applied easily. Through the diversity analysis and simulation results, it is confirmed that diversity order two can be achieved when every node has two antennas and the reliability of IA can be enhanced by the proposed selection scheme of the beamforming matrices.

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초 록

본 논문은 협동통신 네트워크에서 다이버시티 기술과 간섭정렬에 관한 다음 세 가지 연구 결과를 포함하고 있다.

- 분산 시공간 부호를 사용하는 비직교 복호후 전달 프로토콜의 릴레이 온오프 경계
  - 높은 SNR 영역에서 분산 시공간 부호를 사용하는 비직교 복호후 전달 프로토콜의 최적의 릴레이 온오프 경계값 계산.
  - 낮은 SNR 영역에서 분산 시공간 부호를 사용하는 비직교 복호후 전달 프로토콜의 준최적의 릴레이 온오프 경계값 제시
  - 제안된 기법의 다이버시티 오더 분석
- 준정지 $M \times 2$ X 채널을 위한 새로운 릴레이 이용 간섭 정렬 기법 제시
  - 하나의 전이중 (full-duplex) 릴레이를 이용한 간섭 정렬 기법 제시
  - 두 개의 반이중 (half-duplex) 릴레이를 이용한 간섭 정렬 기법 제시
  - 제시된 기법이 준정지 $M \times 2$ X 채널에서 최대 자유도를 얻음을 증명
- MIMO 간섭 채널에서 간섭 정렬의 선택 다이버시티
  - MIMO 간섭 채널에서 간섭 정렬에서 사용될 빔포밍 행렬에 대한 선택 기준 제시

– 제안된 기법의 다이버시티 분석

먼저, 협동 통신 네트워크에서 소스와 릴레이를 이용하여 분산 시공간 부호를 설계한다. 분산 시공간부호를 복호하기 위해, 목적 노드는 선형 결합 복호 기법을 사용한다. 이 시스템 모형에 대한 심볼 오류 확률을 수식화 하고 이를 최소화하는 릴레이 온오프 경계값을 계산한다. 이렇게 계산된 경계값을 이용하여 릴레이 온오프를 할 경우 비직교 복호 후 전달 프로토콜에서 얻을 수 있는 최대 다이버시티 오더를 얻을 수 있다는 것을 증명한다.

본 논문의 두 번째 결과로, 두 개의 릴레이를 이용한 간섭 정렬 기법이 제시된다. 첫번째 간섭 정렬 기법은 하나의 정이중 릴레이를 이용한 것이고, 이를 통해 $M \times 2$ X 채널에서의 최대 자유도를 얻을 수 있다. 그러나 전이중 릴레이는 강력한 자기 간섭 신호로 인해 구현하기가 어렵다. 이 문제를 해결하기 위해 두번째 간섭 정렬 기법에서는 두 개의 전이중 릴레이가 이용되고 이 기법 역시 최대 자유로를 얻는다.

마지막으로, MIMO 간섭 채널에서의 간섭 정렬을 위한 빔포밍 행렬을 선택하는 방법을 제시한다. 대부분의 간섭 정렬의 결과는 자유도에 집중되어 있고, 신뢰도에 대한 중요한 척도인 다이버시티 이득에 대한 결과는 거의 없다. 따라서 본 논문에서는 MIMO 간섭 채널에서 심볼 오류 확률을 최소화하는 빔포밍 행렬의 선택 기준을 제시하고 제안된 기법의 다이버시티 이득을 분석해본다.

주요어: 간섭 정렬, 간섭 채널, 다이버시티 이득, 릴레이, 분산 시공간 블록 부호, 선택 다이버시티, X 채널, 자유도, 협동 통신

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